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Pressure of Pure Neutron Matter and BHFA

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ABSTRACT

This study investigates the pressure of pure neutron matter using the Brueckner–Hartree–Fock Approximation (BHFA), taking into account the effects of three-body forces among nucleons through the introduction of a density-dependent two-body Skyrme-type interaction. The nucleon–nucleon interaction potential of the Nijmegen 1 (Nijm1) type was adopted within the framework of the Brueckner–Hartree–Fock approach to calculate the equation of state and the pressure of pure neutron matter. The theoretical calculations were performed at different temperatures, namely T=0T = 0, 88, and 1212 MeV, in order to investigate the effect of temperature on the pressure behavior as a function of matter density. The obtained results were compared with previous calculations by Baldo and Ferreira using the realistic interaction potential (v14+TNI). The results showed good agreement with previous theoretical predictions, particularly in the low-density region, confirming the capability of the BHFA method, supplemented by an equivalent three-body force correction, to describe the pressure behavior of pure neutron matter satisfactorily. The results indicate that the adopted model provides an appropriate representation of the thermal and mechanical properties of neutron matter within the studied density range, making it a suitable approach for theoretical investigations related to dense nuclear matter physics and the internal structure of neutron stars.

Keywords: Pure neutron matter, pressure, (BHFA).

ضغط المادة النيوترونية النقية وتقريب بروكنر هاتري فوك

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ملخـــــص البحـــــث

يتناول هذا البحث دراسة ضغط المادة النيوترونية النقية باستخدام تقريب بروكنر–هاتري–فوك(BHFA) ، مع الأخذ في الاعتبار تأثيرات القوى الثلاثية بين النيوكليونات من خلال إدخال جهد ثنائي الجسم معتمد على الكثافة من نوع سكيرم. وقد تم اعتماد جهد التفاعل بين النيوكليونات من نوع نجمقن 1 (Nijm1) ضمن إطار تقريب بروكنر–هاتري–فوك لحساب معادلة الحالة وضغط المادة النيوترونية النقية؛ أُجريت الحسابات النظرية عند درجات حرارة مختلفة شملت (T = 0) ، و(8)، و(12) MeV، بهدف دراسة تأثير درجة الحرارة على سلوك الضغط مع تغير كثافة المادة. كما تمت مقارنة النتائج المتحصل عليها مع الحسابات السابقة لكل من بالدو وفيريرا باستخدام جهد التفاعل الواقعي .(v14+TNI) أظهرت النتائج توافقًا جيدًا مع التنبؤات النظرية السابقة، خاصة عند الكثافات المنخفضة، مما يؤكد قدرة منهجية BHFA المدعومة بتصحيح القوى الثلاثية المكافئة على وصف سلوك ضغط المادة النيوترونية النقية بصورة مناسبة؛ وتشير النتائج إلى أن النموذج المستخدم يوفر تمثيلًا ملائمًا للخواص الحرارية والميكانيكية للمادة النيوترونية ضمن نطاق

الكثافات المدروسة، مما يجعله أداة مناسبة للدراسات النظرية المرتبطة بفيزياء المادة النووية الكثيفة وبنية النجوم النيوترونية.

الكلمات المفتاحية: المادة النيوترونية النقية، الضغط، تقريب بروكنر–هاتري–فوك.

1. Introduction

The study of nuclear matter and neutron matter theory aims at reproducing experimentally established properties, such as the free energy, symmetry energies, compressibility. The direct application of Brueckner theory to finite nuclei [1] is not very convenient, therefore effective interactions have been introduced to avoid the complications associated with realistic potentials. In this regard, the Skyrme interaction[2] has proven to be particularly successful. In a specific formulation, the Skyrme force consists of a short - range two-body term together with a zero -range three - body component, which can be represented by a density-dependent two-body interaction [3]. Mansour et al [4] proposed a related approach for neutron matter by employing a term dependent on the densities of spin-up and spin-down neutrons. It is widely recognized that the Skyrme interaction constitutes a straightforward yet effective potential for characterizing the properties of neutron matter. On a microscopic basis, the equation of state of symmetric nuclear matter has been extensively investigated within the variational framework [5-8], as well as through relativistic[9-15] and non-relativistic [16,17] Brueckner-Hartree-Fock (BHF) formalisms. The predictions derived from non-relativistic microscopic approach encompassing both the BHF and variational methods that rely exclusively on two-body nucleon - nucleon (NN) forces (2BF) fail to reproduce the empirical saturation point of symmetric nuclear matter (Coester band[18]). To rectify the nuclear saturation problem, two research avenues have been pursued. The first entails the formulation of relativistic-mean-field (RMF) theory[19] and the Dirac-Brueckner-Hartree-Fock (DBHF) approach[10,20-26]. The DBHF method has proven successful in replicating the saturation characteristics of symmetric nuclear matter (SNM). Nevertheless, several unresolved issues persist, including the negative energy state problem, as well as ambiguities associated with the decomposition of the effective reaction matrix into covariant amplitudes-ambiguities that arise from the various approximations introduced to reduce the four-dimensional Bethe-Salpeter equation to its corresponding three - dimensional form. In the second approach, medium effects are incorporated through either phenomenological interactions or three-body forces (3BF) within non - relativistic frameworks. Calculations employing phenomenological 3BF have been carried out both in the context of the variational method[5-7] and the BHF approximation[27-30]. The fundamental input quantity in BHF calculations is the free space NN interaction. Pure neutron matter (PNM) serves as the theoretical foundation for the equation of state (EOS) of neutron stars. The EOS relates the free energy, energy density, and pressure, which together determine the internal structure of neutron star. Neutron stars are macroscopic realization of the microscopic free energy and pressure of pure neutron matter. At low densities, PNM is relevant for multifragmentation reactions and pion production ratio. In summary, PNM constitutes the theoretical basis for interpreting neutron star observations (including masses, radii, cooling behavior, heavy-ion collision experiments, nuclear structure studies and testing fundamental many - body quantum physics). The objective of the present study is to compute the pressure of pure neutron matter from a theoretical perspective. We employed the modern Nigmegen1 (Nijm1) [31] potential. In this work we include corrections due to three-body forces by means of an equivalent density - dependent two - body force of the Skyrme type. Hot systems are also considered for small temperatures. In the following section we provide a brief outline of the computational method.

2. Material and Methods

Here we start with a short review of the theoretical framework: The microscopic Brueckner-Bethe - Goldstone description of nuclear matter is based on a linked cluster expansion of the energy per nucleon of nuclear matter [30]. The basic ingredient is the Brueckner reaction matrix G, which is the solution of the Bethe–Goldstone equation

$$G(\omega) = V + V\frac{Q}{\omega - H_o + i\eta}G(\omega) \tag{1}$$

Here, ω is the starting energy which is usually the sum of the single -particle energies of the states of the interacting nucleons

$$\omega = e\,(k) + e\,(k'). \tag{2}$$

V is the bare NN potential, η is an infinitesimal small number, H_o is the unperturbed energy of the intermediate scattering states, e is the single-particle energy, and Q is the Pauli projection operator; it projects out states with two nucleons above the Fermi level, it is given by:

$$Q\,(k, k') = (1- \Theta_f\,(k))\,(1- \Theta_f\,(k')), \tag{3}$$

Where $\Theta_f\,(k) = 1$ for $k < k_f$ and zero otherwise, $\Theta_f\,(k)$ is the occupation probability of a free Fermi gas with Fermi momentum k_f. In the Brueckner–Goldstone expansion, the average binding energy per nucleon is expanded in a series of terms as the following

$$\frac{E(k)}{A} = < \hat{T} > + < \hat{G} > = \sum_k \frac{\eta^2 k^2}{2m} + \frac{1}{2}\sum_{k,k'<k_f} < kk'\big|G\big(e(k) + e(k')\big)\big|kk' > \tag{4}$$

where |kk' > refer to antisymmetrized two-body states. This first order is known as the Brueckner–Hartree–Fock (BHF) approximation. To completely determine the average binding energy one has to define the single-particle potential U (k) which contributes to the single particle energies appearing in the G-matrix elements. The structure of the expression (4) suggests choosing the following BHF single-particle potential

$$U(k) = \sum_{k'<k_f} < kk'\big|G\big(e(k) + e(k')\big)\big|kk' > \tag{5}$$

$$\frac{E(k)}{A} = \sum_{k<k_f}\left\{\frac{\eta^2 k^2}{2m} + \frac{1}{2}\,U(k)\right\} = \frac{4}{\rho}\int_0^{kf}\frac{4\pi k^2}{(2\pi)^3}\left(\frac{\eta^2 k^2}{2m} + \frac{1}{2}U(k)\right)dk$$

$$= \frac{3\eta^2 k_f^2}{10m} + \frac{3}{2k_f^3}\int_0^{k_f} k^2\, dkU(k) \tag{6}$$

Where ρ is the matter density. The G-matrix itself depends on U (k) through the starting energy ω, defined in Eq. (2), and the lowest-order approximation (4) along with choice (5) for the single-particle potential is often known as the lowest-order Brueckner theory. The single particle energy e (k) is defined as:

$$e(k) = T + U(k) = \frac{\eta^2 k^2}{2m} + U(k) \quad (7)$$

Where T is the kinetic energy. The conventional choice for the single particle potential has been to take the BHF potential (Eq. (5)) for hole states ($k < k_f$) and zero for particle states ($k > k_f$), thus introducing as:

$$U(k) = \begin{cases} \sum_{k' \leq kf} < kk' \left| G\left(e(k) + e(k')\right) \right| kk' > & k \leq k_f \\ 0 & k > k_f \end{cases} \quad (8)$$

Eqs. (1) and (7) represent the main equations that one has to solve self-consistently. In order to achieve saturation in nuclear matter one has to add three - body interaction terms or a density dependent two - nucleon interaction. We have chosen it following the notation of the Skyrme interaction to be of the form

$$v(\boldsymbol{r}_1, \boldsymbol{r}_2) = \sum_i t_i (1 + y_i p_\sigma)\, \rho^{\alpha_i}\, \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \quad (9)$$

Where $\mathbf{r_1}$ and $\mathbf{r_2}$ are the position vectors for the particle 1 and particle 2 respectively, P_σ is the spin exchange operator, ρ is the matter density t_i, y_i and α_i are parameters. For various values of α_i (typically $\alpha_i = 1/3$, 2/3, 1/2, and 1) we have fitted t_i and y_i in such a way that a BHF calculation plus the contact terms yield the empirical saturation point for symmetric nuclear matter. Having obtained the energy per particle E/A for zero temperature, the free energy and the pressure may be calculated at temperature T using the expression of the level density[32]. Among the different sets of parameters α_i proposed here the best results were obtained for two terms of the above summation where α_1 and α_2 are equal to 1/3 and 2/3 respectively. A comparison is made with M. Baldo and L. S. Ferreira calculation [33] v 14+TNI realistic potential. The results are identical with M. Baldo and L. S. Ferreira at low densities.

3. Results and Discussion

Calculation of the pressure

$$F = E_{T=0} - a\, T^2 \quad (10)$$

$$a = \left(\frac{\pi^2}{2}\right)\left(\frac{m^*}{\hbar^2 k_f^2}\right) \quad (11)$$

$$k_f^2 = (3\pi^2 \rho)^{\frac{2}{3}} \quad (12)$$

Where F is the free energy of the system, $E_{T=0}$ is the total energy at T=0, a is the level density of the system, ħ is the Plank's constant divided by 2π and m^* is the effective mass of the nucleon and k_f is the Fermi momentum. By differentiating equation (10) we get on the pressure of neutron matter as the following

$$p = p_{T=0} + \left(\frac{\pi^2 m^* k_f^2 T^2}{6\, \hbar^2}\right) \quad (13)$$

The equation (13) is used to calculate the pressure of the pure neutron matter. Where P is the pressure of the system, $P_{T=0}$ is the total pressure at T=0, ħ is the Planck's constant divided by 2π and m* is the effective mass of the neutron. The results are shown in the Fig. 1 at T=0, Fig. 2 at T= 8 MeV and Fig. 3 at T =12 MeV in comparison with M. Baldo and L. S. Ferreira calculation [33].

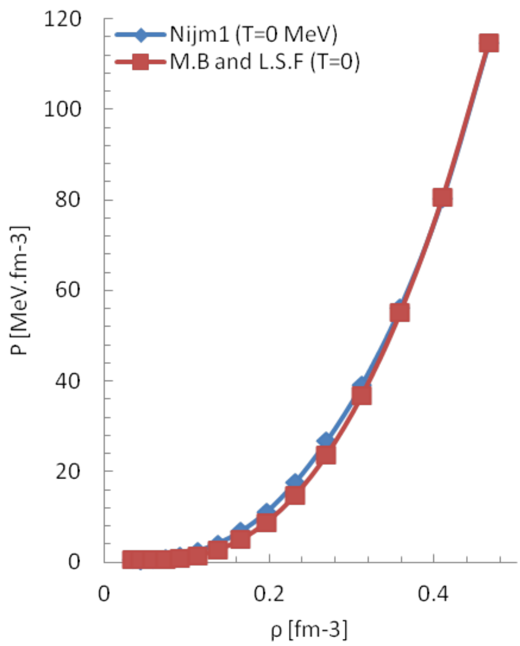

Fig.1: Pressure of the pure neutron matter in [MeV.fm^{-3}] at (T=0) as a function of density in [fm^{-3}] using Nijm1 potential in comparison with M. B and L. S. F [33].

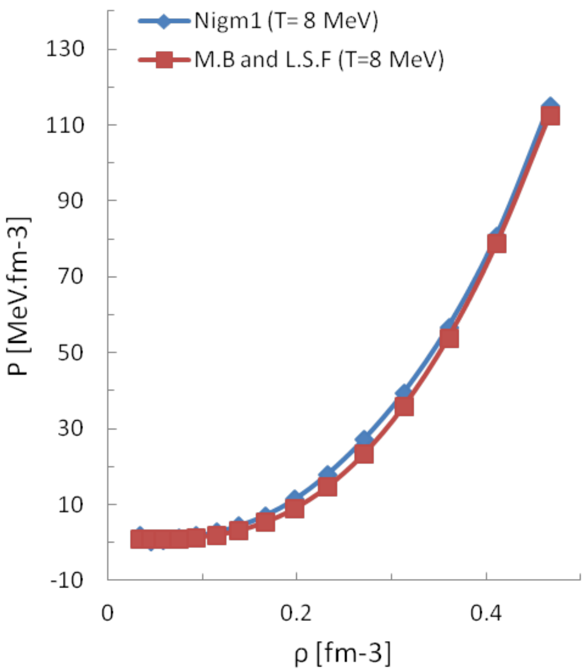

Fig.2: Pressure of the pure neutron matter in [MeV.fm^{-3}] at (T=8MeV) as a function of density in [fm^{-3}] using Nijm1 potential in comparison with M. B and L. S. F [33].

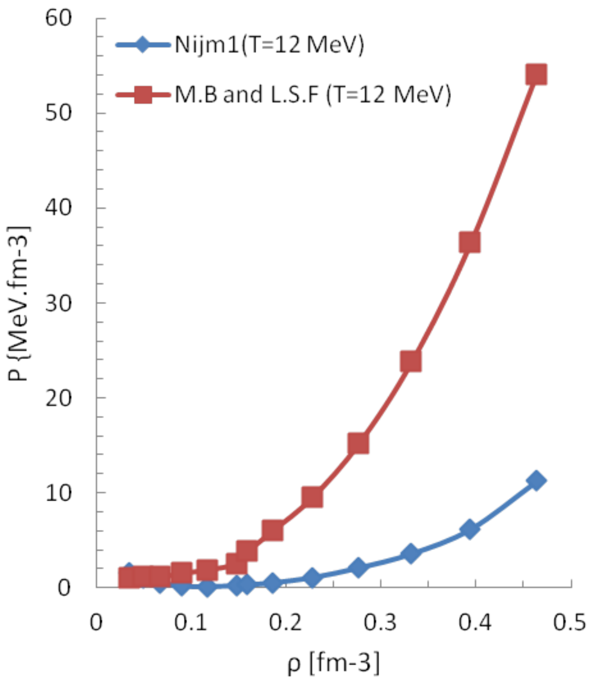

Fig.3: Pressure of the pure neutron matter in [MeV.fm^{-3}] at (T=12MeV) as a function of density in [fm^{-3}] using Nijm1 potential in comparison with M. B and L. S. F [33].

4. Conclusion:

In this work we have computed the pressure of pure neutron matter at temperatures T=0, 8 MeV and 12 MeV. The obtained results were derived by incorporating a density-dependent two - body Skyrme interaction into the Brueckner - Hartree - Fock (BHF) formalism. The modern nucleon-nucleon (NN) interaction, represented by the Nijm1 potential was employed within the BHF approximation framework. The calculated results were compared with those obtained using the realistic v_{14} + TNI interaction by M. Baldo and L. S. Ferriera[33]. The study demonstrates that the BHF approach combined with the proposed contact interaction is capable of reproducing the pressure behavior of pure neutron matter with satisfactory agreement relative to the realistic force calculations reported by M. Baldo and L. S. Ferreira. The pressure values for pure neutron matter at T = 0, 8 MeV and 12 MeV were successfully evaluated, and the findings exhibit good consistency with the corresponding theoretical results available in the literature. It is inferred that the applicability of these calculations is mainly restricted to the low density region. Overall, the adopted method yields results that are favorable agreement with previous theoretical predictions**.**

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