Comparative Performance Analysis for Transient and Steady State Response of DC Motor with One and Two Degree of Freedom Phase Lag-Lead Compensators using Frequency Response Technique

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ABSTRACT

This paper introduces design of different compensators and their output response for speed control of Direct Current (DC) motor with One and Two Degree of Freedom. DC motors are used in many industrial and commercial applications require higher performance, reliability, variable speed, easier controlling stability, accuracy, speed and position control of motor is required. In this paper in order to simulate the approach a MATLAB/SIMULINK model having one and two degree of freedom phase lead, phase lag and phase lead-lag compensators are constructed to control the modeled Direct Current (DC) motor for enhancement of static & dynamic response. In addition to that, the effect of adding two degree of freedom compensators on the transient and steady state response of the system is studied.

Keywords: DC Motor, Phase Lead Compensator, Phase Lag Compensator, Phase Lag-Lead, MATLAB/Simulink etc.

I. INTRODUCTION

DC motor has been popular in the industry control area for a long time because they have good characteristics that is high starting torque characteristics, high response performance and easier to be linear control. These motors are commonly used to provide rotary (or linear) motion to a variety of electromechanical devices and servo systems. DC motor has been widely used in industry even though its maintenance costs are higher than the induction motor. DC motor has good control response, wide speed control range and it is widely used in systems which need high control requirements, such as rolling mill, double-hulled tanker, high precision digital tools, etc. There are several well-known methods to control DC motors such as Compensation Technique and Controller Method. Despite a lot of researches and the huge number of different solutions proposed. The purpose of developing a control system is to enable stable and reliable control. Once the control system has been specified and the type of control has been decided, then the design and analysis are done. There are three major objectives of system analysis and design: producing the desired transient response, reducing steady-state error, and achieving stability. The control system has overall response that is transient response and steady state response is analyzed and to evaluate the performance of the optimal model of DC motor time response analysis and frequency response analysis are obtained [1]. The purpose of this paper is to comparative performance analysis for Transient and Steady State Response of DC Motor with One and Two Degree of Freedom Phase Lag-Lead Compensators using Frequency Response Technique (Bode Plot) for improving the transient and steady state response is presented. In addition to that, the effect of adding two degree of freedom compensators on the Transient and Steady State Response of the system is studied. In the frequency-response approach, the transient-response performance is specified in an indirect manner. The frequency domain specifications can be conveniently met in the Bode diagram approach. Therefore the open loop has been designed by the frequency-response method, the closed loop poles and zeros can be determined. The transient-response characteristics must be checked to see whether the designed system satisfies the requirements in the time domain. If it does not, then the compensator must be modified and the analysis repeated until a satisfactory result is obtained. Design in the frequency domain is simple and straightforward. The frequency-response plot indicates clearly the manner in which the system should be modified. In order to obtain the desired performance of the system, compensating networks are used. It is a fundamental building block in classical control theory. There are two general types of compensators: Lead compensators, and lag Compensators. If the two types are combined, a special lag-lead Compensator system can be gotten. Compensators influence disciplines as varied as robotics, satellite control, automobile diagnostics, and laser frequency stabilization. They are an important building block in analog control systems, and can also be used in digital control [2]. This paper is organized as follows. Mathematical Modeling of the DC Motor is given in Sec. II. Design Procedure of Compensators using Frequency Response Method (Bode Plot) is given in Sec. III. Analysis of Simulation Results is given in Sec. IV. Conclusion is demonstrated in Sec. V.

II. MATHEMATICAL MODELING OF THE DC MOTOR

The mathematical model of the DC motor is fundamental for the corresponding performance analysis and control system design. Fig. 1 shows a DC motor [3].

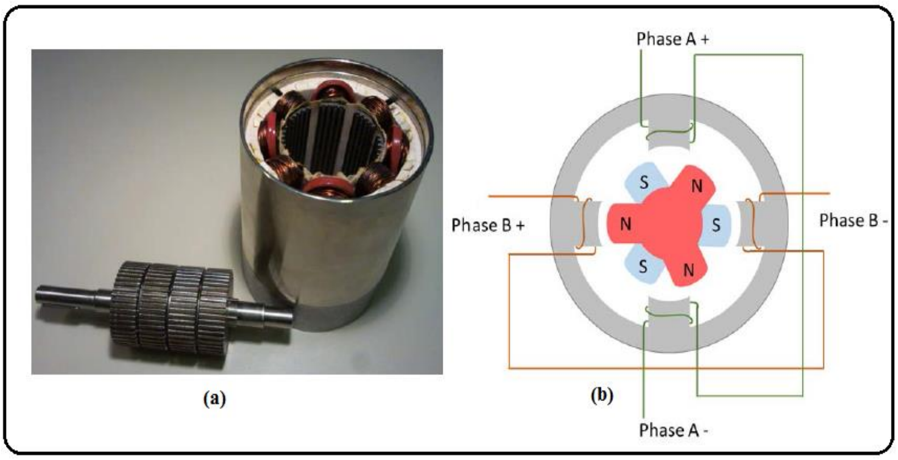

Figure 1: DC Motor (a) separated rotor and stator and (b) with three teeth [3].

The DC motor model could be divided into two sub-models, an electrical and a mechanical model. They are presented as follow:

A. The Electrical model of The DC motor

DC motor could be modeled as an RL circuit plus a back electromotive force (emf) in Fig. 2. According to [3] the differential equations of phase *a* and phase *b* are given by equation (1) and (2), respectively,

$$L_a \frac{di_{a(t)}}{dt} = -R_a i_a(t) - e_a(t) + v_a(t) \qquad (1)$$

$$L_b \frac{di_{b(t)}}{dt} = -R_b i_b(t) - e_b(t) + v_b(t) \qquad (2)$$

Where

R_a and R_b : → are the phase resistances
L_a and L_b : → are the phase inductances
i_a and i_b : → are the phase currents
v_a and v_b : → are the terminal voltages

And the back emf voltages are described by:

$$e_a(t) = -K_m \omega_m \sin(\rho\theta_m) \qquad (3)$$

$$e_b(t) = K_m \omega_m \cos(\rho\theta_m) \qquad (4)$$

Where

Km : → is the motor constant.
p : → is the number of motor pole pairs (teeth).
ω_m: →is the rotor (mechanical) angular speed [rad/s].
θ_m : → is the rotor (mechanical) angular position [rad].

In equation (1) and (2), the phase resistances and inductances are assumed to be equal, $R_a = R_b = R$ [ohm], $L_a = L_b = L$ [H]. Moreover, $v_a(t)$ and $v_b(t)$ are the terminal voltages [V], $e_a(t) = e_b(t)$ are the back emf [V].

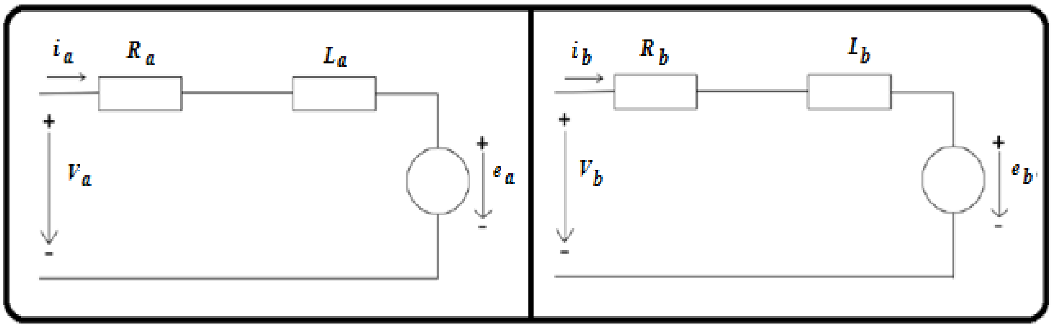

Figure 2: The equivalent circuits DC motor.

B. The Mechanical model of DC motor

The shaft of the DC motor, which represents the mechanical part of the system, is modeled as a rigid body subjected to different torques as shown in Fig. 3 [9].

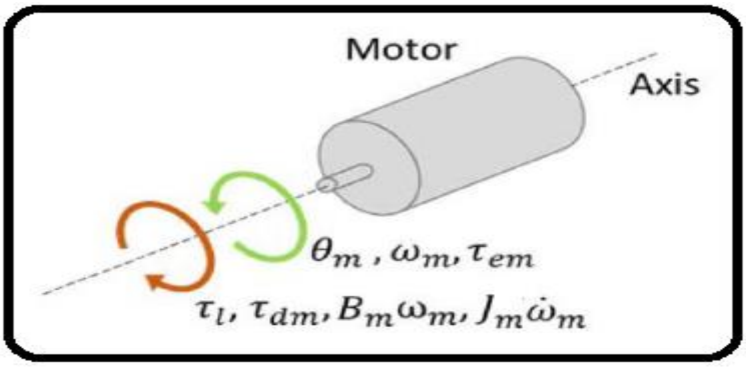

Figure 3: The mechanical sub-system of the DC motor [9].

The differential equation of the mechanical sub-system is given by equation (5),

$$J_m \frac{d\omega_m}{dt} = \tau_{em} - B_m\omega_m - \tau_{dm} - \tau_l \qquad (5)$$

And

$$\tau_{em} = K_m(-i_a \sin(\rho\theta_m) + i_b \cos(\rho\theta_m)) \qquad (6)$$

$$\tau_{dm} = T_{dm} \sin(2\rho\theta_m + \emptyset) \qquad (7)$$

Where

τ_{em} : →is the motor's electromagnetic torque or the generated torque [*N.m*]

J_m : → is the motor moment of inertia [*kg.m*^{2}]

B_m: → is the motor viscous friction coefficient [*kg/s.m*]

T_{dm}: → is the detent torque amplitude applied [*N.m*]

$\emptyset$: → is a phase shift related to τ_{dm}

τ_l: →is the external or applied load torque [*N.m*]

With the following relation:

$$\theta_e = \rho\theta_m \qquad (8)$$

Where

θ_e: →is the electrical rotor position

III. DESIGN PROCEDURE OF COMPENSATORS USING FREQUENCY RESPONSE METHOD.

A. Frequency Response Approach (Bode Plot)

A compensator is added to a system to improve its steady state as well as dynamic response. Nyquist plot is difficult to modify after introducing compensator. Instead, Bode plot is used since two important design criteria, phase margin and gain crossover frequency are visible from the Bode plot along with gain margin. In Bode plot, low frequency asymptote of the magnitude curve is indicative of one of the error constant ***[Kp], [Kv], [Ka]*** depending on the system types. In Bode Plot, specifications on the transient response can be translated into phase margin [***P.M***], gain margin[***G.M***], gain crossover frequency, bandwidth etc. Design using Bode plot is simple and straight forward and reconstruction of Bode plot is not a difficult task.

B. Phase Lead Compensator Design using Bode Plot

The phase lead compensator is used to improve stability margin and increases system bandwidth thus improving the spread of the response [4-6]. The transfer function of the phase lead compensation network is

$$G_{Lead}(s) = K\left(\frac{1+sT}{1+\alpha sT}\right) = K_c\alpha\left(\frac{1+sT}{1+\alpha sT}\right) \qquad (9)$$

Where

α : → is an attenuation factor hence lead compensator is always used with an amplifier

[$1/\alpha$], [$0 < \alpha < 1$].

T : → is the time constant, [$T > 0$].

Procedures for Design of Phase Lead Compensator using Frequency Response Techniques (Bode plot) are as follows:

1- Determine the phase lead compensator gain [$K=K_c\,\alpha$] satisfying the given error constant (according to steady state error requirement).

2- Draw the Bode plot for the uncompensated system ***Gp(s)*** after introducing [$K=K_c\,\alpha$] in the system, which means [***KGp(s)***], and compute the gain crossover frequency [ω_g] and the phase margin [***P.M***].

3- Determine the amount of phase lead angle to be contributed by the phase lead compensating network by using the formula

$$\phi_m = \phi_d - \phi_u - \epsilon \qquad (10)$$

Where

Φ_m : → is the maximum phase lead angle of the phase lead compensator

Φ_d : → is the desired phase margin [phase margin specified or necessary phase margin required to be added]

Φ_u : → is the phase margin [***P.M***] of uncompensated system [***KGp(s)***]

ϵ:→ is the additional phase lead angle to compensate for shift in gain crossover frequency [ω_g] (margin of safety from 5^0 to 15^0 or ***(+10% ~*** 15%)]

4-Calculate or obtain [α] from the required [Φ_m] by using the equation

$$\alpha = \frac{1 - \sin(\phi_m)}{1 + \sin(\phi_m)} \qquad (11)$$

5- From the Bode plot, determine the frequency at which the magnitude of the uncompensated system [***KGp(s)***] is [$-20\log_{10}(1/\sqrt{\alpha})$]. This frequency is the new gain crossover frequency [$\omega_{g_{new}}$] where the maximum phase lead angle [Φ_m] should occure

$$K\left|G_P(j\omega_{g_{new}})\right| = -20\log_{10}\left(\frac{1}{\sqrt{\alpha}}\right) \qquad (12)$$

6- Make

$$\omega_m = \omega_{g_{new}} \qquad (13)$$

7- Calculate [***T***] from the Relation

$$\omega_m = \frac{1}{T\sqrt{\alpha}} \quad \Longrightarrow \quad T = \frac{1}{\omega_m\sqrt{\alpha}} \quad (14)$$

8- Find the transfer function of phase lead compensator [$G_{lead}(s)$]

$$G_{Lead}(s) = K\left(\frac{1 + sT}{1 + \alpha sT}\right) = K_c\alpha\left(\frac{1 + sT}{1 + \alpha sT}\right) \quad (15)$$

9- Determine the open-loop transfer function of compensated system

$$G(s) = G_{Lead}(s)G_P(s) = K_c\alpha\left(\frac{1 + sT}{1 + \alpha sT}\right)G_P(s) \quad (16)$$

10- Draw the Bode plot of the compensated system and verify if the design satisfies the specifications. If the phase margin of the compensated system [***G(s)***] is less than the required phase margin then repeat steps[***3 to 10***] by taking [ϵ] as [5^0] more than the previous design.

C. Phase lag compensator design using Bode Plot

The essential feature of a Phase lag compensator is to provide an increased low frequency gain, thus decreasing the steady state error, without changing the transient response significantly [7]. For frequency response design, it is convenient to use the following transfer function of the phase lag compensator

$$G_{Lag}(s) = K\left(\frac{1 + sT}{1 + s\beta T}\right) = K_c\beta\left(\frac{1 + sT}{1 + s\beta T}\right) \quad (17)$$

Where

$\boldsymbol{T}$ and $\boldsymbol{\beta}$: → are respectively the time constant and DC gain, $[\boldsymbol{T} > \mathbf{0}]$, $[\boldsymbol{\beta} > \mathbf{1}]$.

Procedures for Design of Phase Lag Compensator using Frequency Response Techniques (Bode plot) are as follows:

1- Determine the phase Lag compensator gain [$K=K_c\ \alpha$] satisfying the given error constant (according to steady state error requirement).

2- From the Bode plot, determine the phase margin of the uncompensated system

$$\boldsymbol{\phi_u = \phi_s - \epsilon} \qquad (18)$$

Where

Φ_s : → Specified or desired phase margin of the uncompensated system

ϵ: → Margin of safety [In ***general*** 5^0 ***to*** 15^0]

Φ_u:→ The phase margin [***P.M***] of uncompensated system [***KGp(s)***]

Since the [***P.M***] is achieved only by selecting [K_c]. It might be deviated from this value when the other parameters are also designed. Thus a safety margin is putted.

3- Determine the frequency corresponding to the specified or required phase margin [Φ_s] plus safety margin [ϵ], which means [Φ_u=$\Phi_s + \epsilon$] from the phase curve. This frequency is new gain crossover frequency [$\omega_{g_{new}}$]

$$\boldsymbol{P.M = \phi_u = 180^0 + \angle K_c G_P(j\omega_{g_{new}})} \qquad (19)$$

Where

$\omega_{g_{new}}$: → New gain crossover frequency [rad/sec].

$K_c G_P(j\omega_{g_{new}})$: → The transfer function of the uncompensated system with new gain crossover frequency.

4- The magnitude curve is brought down to [***0 db***] at the new gain crossover frequency [$\omega_{g_{new}}$] where the phase margin is satisfied, the phase Lag network must provide the amount of attenuation equal to the value of magnitude curve at [$\omega_{g_{new}}$].

$$\boldsymbol{K_c\left|G_P(j\omega_{g_{new}})\right| = 1 \quad or \quad -K_c\beta\left|G_P(j\omega_{g_{new}})\right| = -20\log_{10}(\beta)} \qquad (20)$$

5- Calculate or obtain [β] by using the equation

$$K = K_c\beta \Rightarrow \beta = \frac{K}{K_c} \quad or \quad \beta = 10^{\frac{|G_P(j\omega_{g_{new}})|}{20}} \qquad (21)$$

6- Calculate [***T***] from the Relation, the only parameter left to be designed is [T]

$$\frac{1}{T} = \frac{\omega_{g_{new}}}{10} \Rightarrow T = \frac{10}{\omega_{g_{new}}} \qquad (22)$$

Usually the upper corner frequency [***1/T***] is placed at a frequency about one decade below the new gain crossover frequency [$\omega_{g_{new}}$]

7- Find the transfer function of phase Lag compensator [$G_{lag}(s)$]

$$G_{Lag}(s) = K\left(\frac{1+sT}{1+s\beta T}\right) = K_c\beta\left(\frac{1+sT}{1+s\beta T}\right) \qquad (23)$$

8- Determine the open-loop transfer function of compensated system

$$G(s) = G_{Lag}(s)G_P(s) = K_c\beta\left(\frac{1+sT}{1+s\beta T}\right)G_P(s) \quad (24)$$

9- Draw the Bode plot of the compensated system and investigate to see if the required phase margin is met or not, if not adjust the value of [β] and [T].

D. Phase lag-lead compensator design using Bode Plot

Lag-Lead compensator is an electrical network which produces phase lag at one frequency region and phase lead at other frequency region. It is a combination of both the lag and the lead compensators. When a single phase Lead or phase Lag compensator cannot guarantee the specified design criteria, a phase Lag-Lead compensator is used. The speed of response and steady state error can be simultaneously improved if both phase Lead and Phase Lag compensation networks are used. Therefore, phase lag-lead compensator is employed where fast time response as well as better steady state accuracy is desired [8-12]. The transfer function of the phase lead compensation network is

$$G_{Lag-Lead}(s) = K\left(\frac{1+sT_1}{1+s\beta T_1}\right)\left(\frac{1+sT_2}{1+\alpha sT_2}\right) \qquad (25)$$

Where

$$[\beta > 1], \quad and \quad [0 < \alpha < 1]$$

Procedures for Design of Phase Lag-Lead Compensator using Frequency Response Techniques (Bode plot) are as follows:

1- Determine the phase Lag-lead compensator gain [***K***] satisfying the given error constant (according to steady state error requirement).

2- Draw the Bode plot for the uncompensated system ***Gp(s)*** after introducing [***K***] in the system, which means [***KGp(s)***], and compute the gain crossover frequency [ω_g] and the phase margin [$P.M=\Phi_u$].

3- Determine the amount of phase Lead angle to be contributed by the phase Lead compensating network by using the formula

$$\phi_m = \phi_d - \phi_u - \epsilon \qquad (26)$$

Where

Φ_m : → is the maximum phase lead angle of the phase lead compensator
Φ_d : → is the desired phase margin [phase margin specified or necessary phase margin
required to be added]
Φ_u : → is the phase margin [***P.M***] of uncompensated system [***KGp(s)***]
ϵ: → is the additional phase lead angle to compensate for shift in gain crossover
frequency [ω_g] (margin of safety from 5^0 to 15^0 or ***(+10% ~ 15%)***)]

4- Calculate or obtain [α] from the required [Φ_m] by using the equation

$$\alpha = \frac{1 - \sin(\phi_m)}{1 + \sin(\phi_m)} \qquad (27)$$

5- From the Bode plot, determine the frequency at which the magnitude of the uncompensated system [***KGp(s)***] is [$-20\log_{10}(1/\sqrt{\alpha})$]. This frequency is the new gain crossover frequency [$\omega_{g_{new}}$] where the maximum phase lead angle [Φ_m] should occurs

$$K\left|G_P(j\omega_{g_{new}})\right| = -20\log_{10}\left(\frac{1}{\sqrt{\alpha}}\right) \qquad (28)$$

6- Make

$$\omega_m = \omega_{g_{new}} \qquad (29)$$

7- Calculate [T_2] from the Relation

$$\omega_m = \frac{1}{T_2\sqrt{\alpha}} \quad \Longrightarrow \quad T_2 = \frac{1}{\omega_m\sqrt{\alpha}} \qquad (30)$$

8- Determine the parameter [β] from the relation

$$-20\log_{10}\left|KG_{Lag}(j\omega_{g_{new}})G_P(j\omega_{g_{new}})\right| = -20\log_{10}(\beta) \qquad (31)$$

9- Calculate [T_1] from the Relation

The only parameter left to be designed is [T_1]

$$\frac{1}{T_1} = \frac{\omega_{g_{new}}}{10} \Longrightarrow T_1 = \frac{10}{\omega_{g_{new}}} \qquad (32)$$

[$1/T_1$] Should be placed much below the new gain crossover frequency [$\omega_{g_{new}}$] to retain the desired value [***P.M***]

10- Find the transfer function of phase Lag-lead compensator [$G_{Lag\text{-}lead}(s)$]

$$G_{Lag-Lead}(s) = K\left(\frac{1+sT_1}{1+s\beta T_1}\right)\left(\frac{1+sT_2}{1+\alpha sT_2}\right) \qquad (33)$$

11- Determine the open-loop transfer function of compensated system

$$G(s) = G_{Lag-Lead}(s)G_P(s) = K\left(\frac{1+sT_1}{1+s\beta T_1}\right)\left(\frac{1+sT_2}{1+\alpha sT_2}\right)G_P(s) \quad (34)$$

12- Draw the Bode plot of the compensated system [***G(s)***] and verify if the specifications are satisfied or not [13].

IV. ANALYSIS OF SIMULATION RESULTS.

The modeling of the DC motor with different types of one and two degree of freedom compensators has been derived. in addition to that, simulation and performance analysis of the DC motor with and without compensators (one and two degree of freedom phase lead compensator, phase lag compensator, and phase lag-lead compensator) have been implemented and investigated by using MATLAB/SIMULINK software.

A. Design Requirements of the control system

The goal of control engineering design is to obtain the configuration, specifications, and identification of the key parameters of a proposed system to meet an actual need. The design process is arranged into three groups [14-15]:

- Establishment of goals and variables to be controlled, and definition of specifications (metrics) against which to measure performance.
- System definition and modeling.
- Control system design and integrated system simulation and analysis.

The reference input is simulated by unit step input, then the actual output response of the DC motor should have the design requirements for the System in terms of time response specifications as shown on Table 1 and the design requirements for the System in terms of frequency response specifications as shown on Table 2.

Table 1: *Design requirements of the system in terms of the Time respons (Transient and steady state response)*

Time Domain Specifications	Design requirements of the system
Settling Time (t_s)	< 0.5 sec or < 500 ms
Maximum Overshoot (M_p)	< 5%
Peak Time (t_p)	< 0.15 sec or < 150 ms
Rise Time (t_r)	< 0.1 sec or <100 ms
Delay Time (t_d)	< 0.05 sec or <50 ms
Damping ratio (ζ)	$0.65 < \zeta < 0.75$
Steady state error (e_{ss})	< 0.02 or 2%

Table 2: *Design requirements of the system in terms of the Frequency response*

Frequency Domain Specifications	Design requirements of the system
Phase Margin (P.M)	Positive and , at least $\geq 60^0$
Gain Margin (G.M)	Positive and large, at least $\geq$ 10 (dB)
Bandwidth (ω_b)	Large as can possible
Resonant Peak (M_r)	Small as can possible

	[$1 \le M_r \le 1.4$] or [$0\ dB \le M_r \le 3\ dB$]
Resonant Frequency (ω_r)	Large as can possible
Cut-off Frequency (ω_c)	Large as can possible
Gain crossover Frequency (ω_{gc})	It must be $\omega_{gc} < \omega_{pc}$ (Hz)
Phase crossover Frequency(ω_{pc})	It must be $\omega_{gc} < \omega_{pc}$ (Hz)

B. Simulation and analysis of the DC motor with One and Two Degree of Freedom Phase Lead Compensator using Bode plot

The block diagram of the DC motor with one degree of freedom phase Lead compensator is shown in Fig. 4.

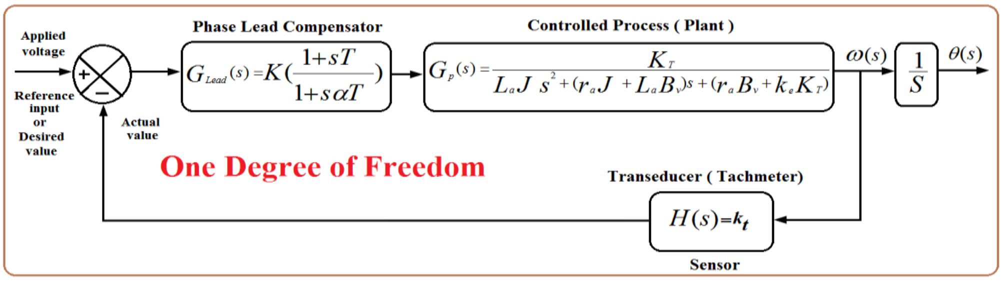

Figure 4: The block diagram of the DC motor with One Degree of Freedom phase Lead compensator

Where

K_t : → is the tachometer constant

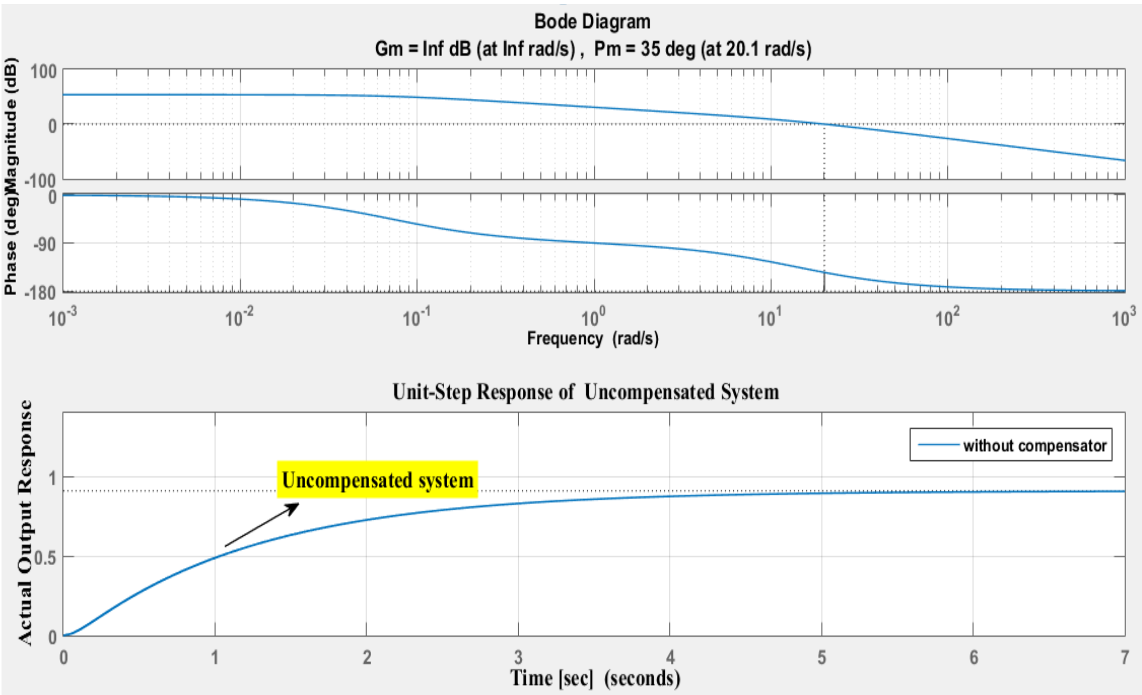

Figure 5: Simulation of the DC motor by unit step response and Bode plot

without Phase Lead Compensator

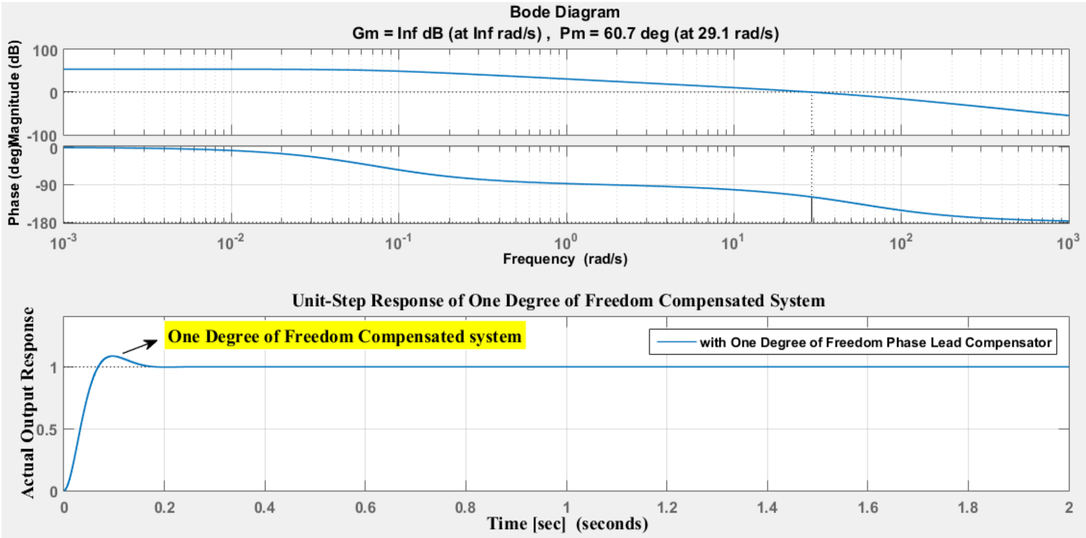

Figure 6: Simulation of the DC motor by unit step response and Bode plot with One Degree of Freedom phase lead compensator

The block diagram of the DC motor with Two Degree of Freedom phase Lead compensator is shown in Fig. 7

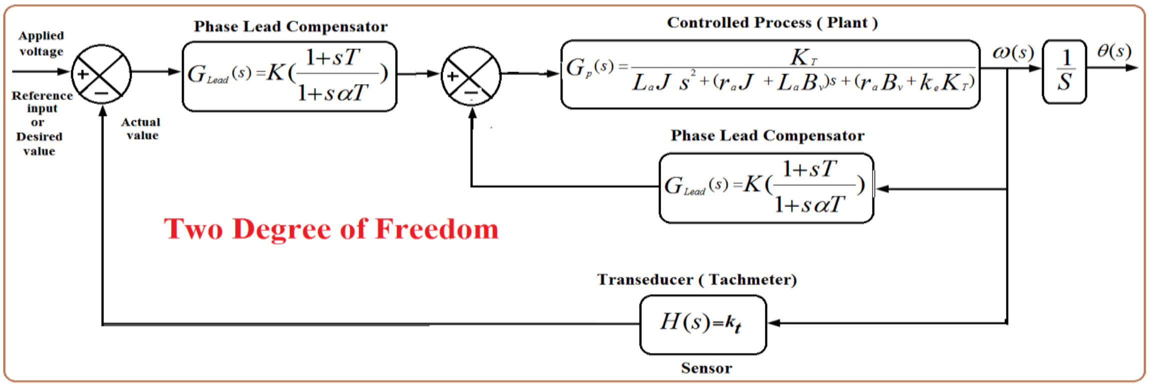

Figure 7: The block diagram of the DC motor with Two Degree of Freedom phase Lead compensator

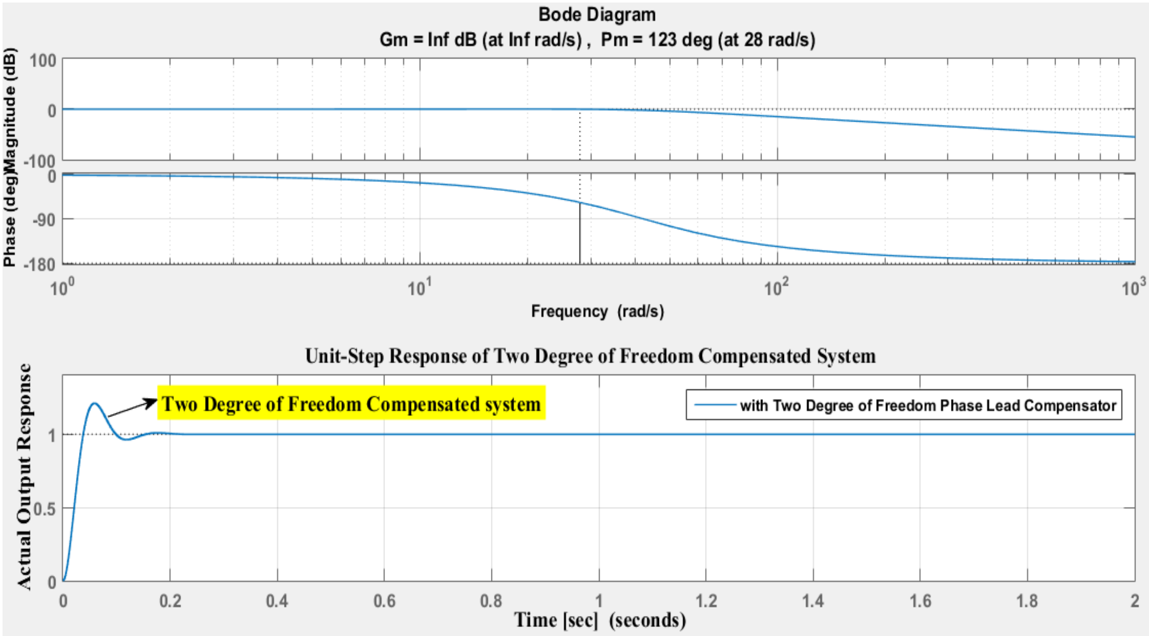

Figure 8: Simulation of DC motor by unit step response and Bode plot with Two Degree of Freedom phase lead compensator

From the Fig. 5 , Fig. 6 and Fig. 8, it is noted that, actual output response of the DC motor with one and two degree of freedom phase lead compensator (compensated system) is better than actual output response of the DC motor without phase lead compensator (Uncompensated system), but still some of design requirements of the systems are not satisfied. Therefore, the another type of compensator is needed to get all design requirements of the system.

C. Simulation and analysis of the DC motor with One and Two Degree of Freedom Phase Lag Compensator using Bode plot

The block diagram of the DC motor with One Degree of Freedom phase Lag compensator is shown in Fig. 9.

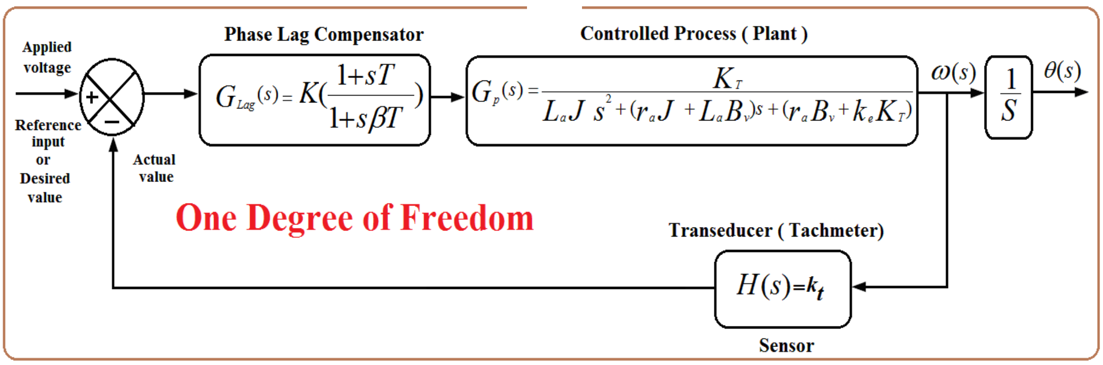

Figure 9: The block diagram of the DC motor with One Degree of Freedom phase Lag compensator

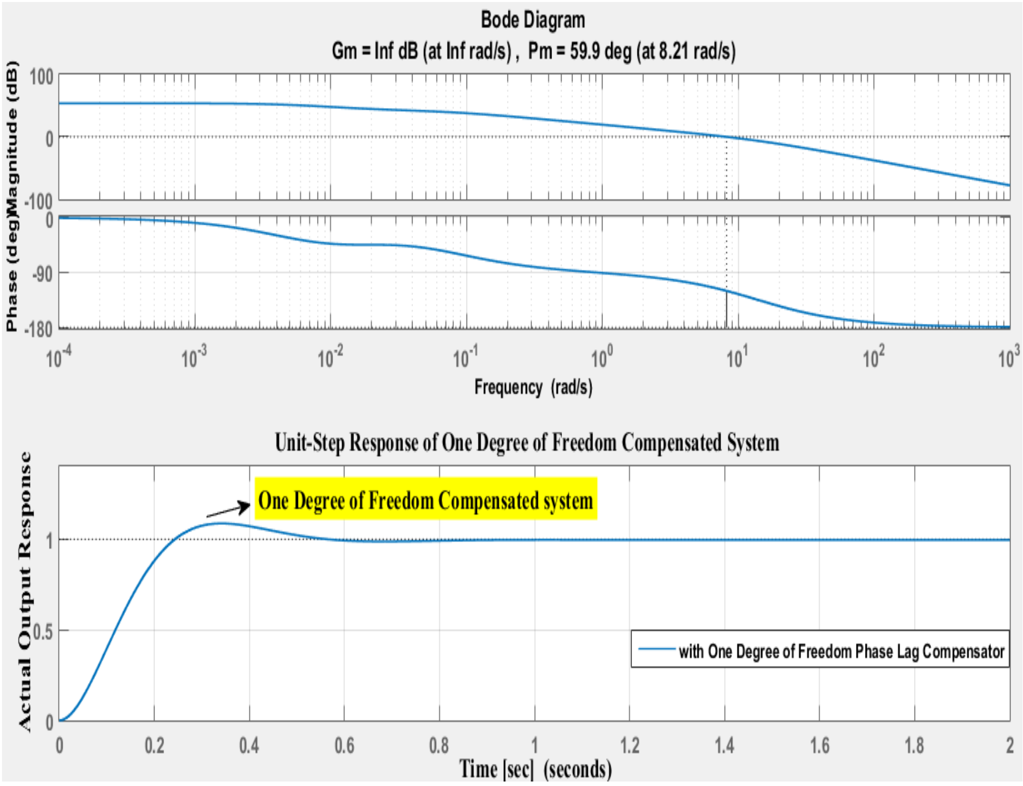

Figure 10: Simulation of the DC motor with One Degree of Freedom phase lag compensator in addition to Bode plot

The block diagram of the DC motor with Two Degree of Freedom phase Lag compensator is shown in Fig. 11.

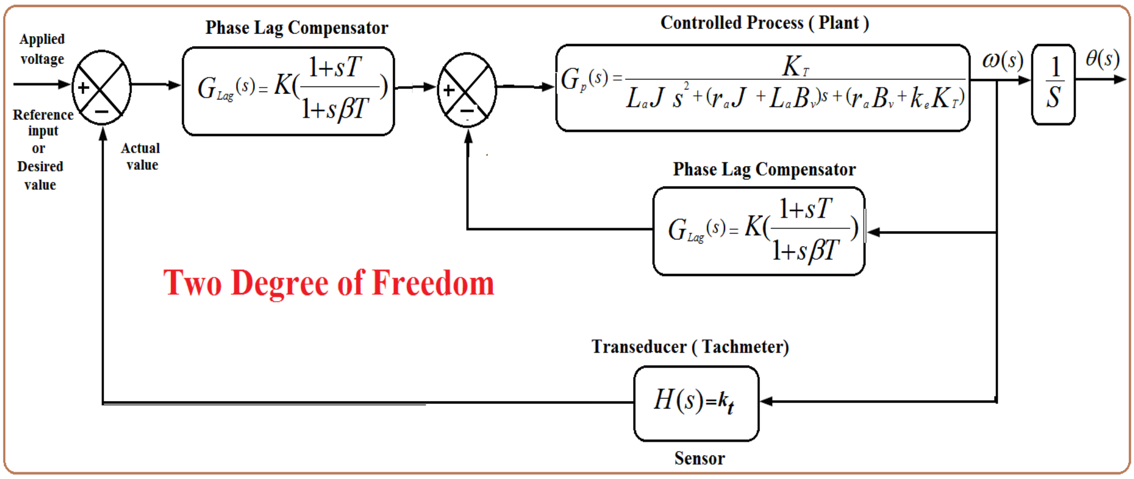

Figure 11: The block diagram of the DC motor with Two Degree of Freedom phase Lag compensator

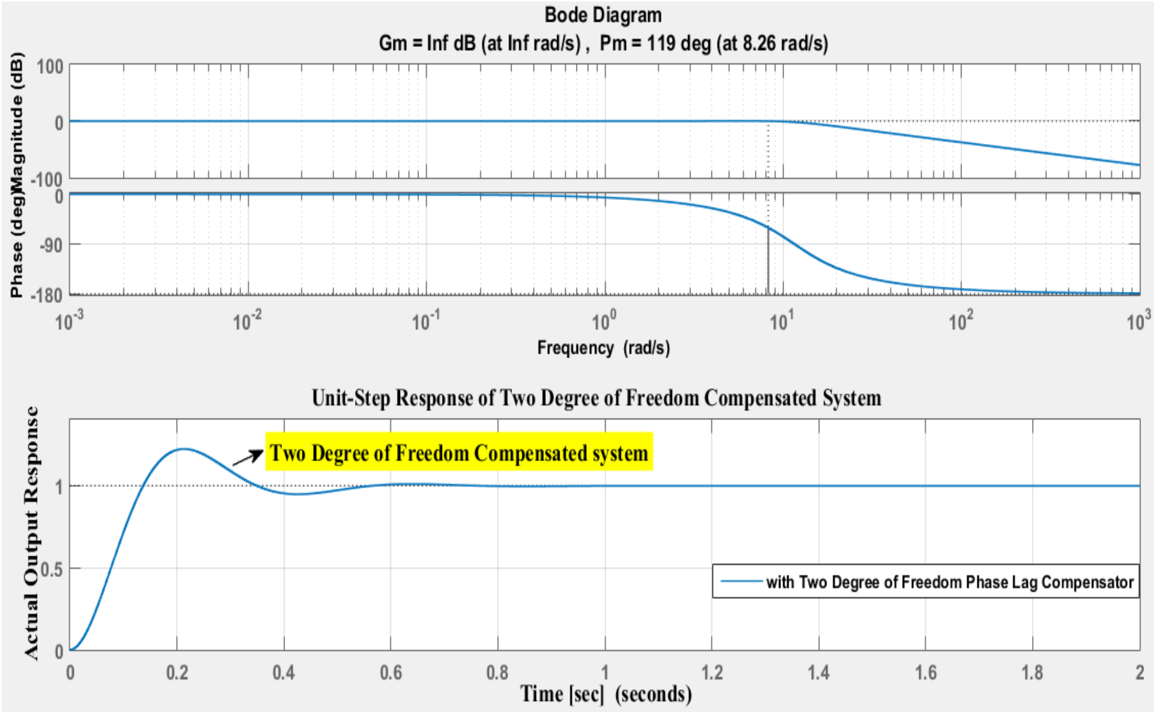

Figure 12 Simulation of the DC motor with Two Degree of Freedom phase lag compensator in addition to Bode plot

From the Fig. 10 and Fig. 12, it is noted that that, actual output response of the DC motor with One and Two Degree of Freedom phase lag compensator (Compensated system) is better than actual output response of the DC motor without phase lag compensator (Uncompensated system), but still some of design requirements of the systems are not satisfied and fulfilled. Therefore, the another type of compensator is needed to get all design requirements of the system.

D. Simulation and analysis of the DC motor with One and Two Degree of Freedom Phase Lag-Lead Compensator using Bode plot

The block diagram of the DC motor with one degree of freedom phase Lag-Lead compensator is shown in Fig. 13.

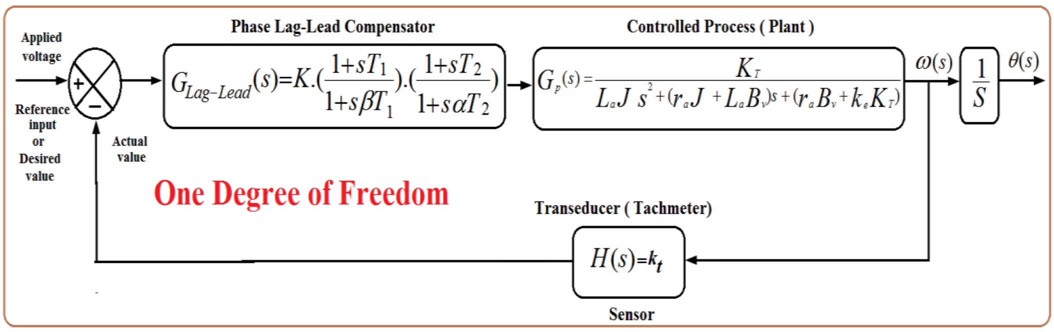

Figure 13: The block diagram of the DC motor with One degree of freedom phase Lag-Lead compensator

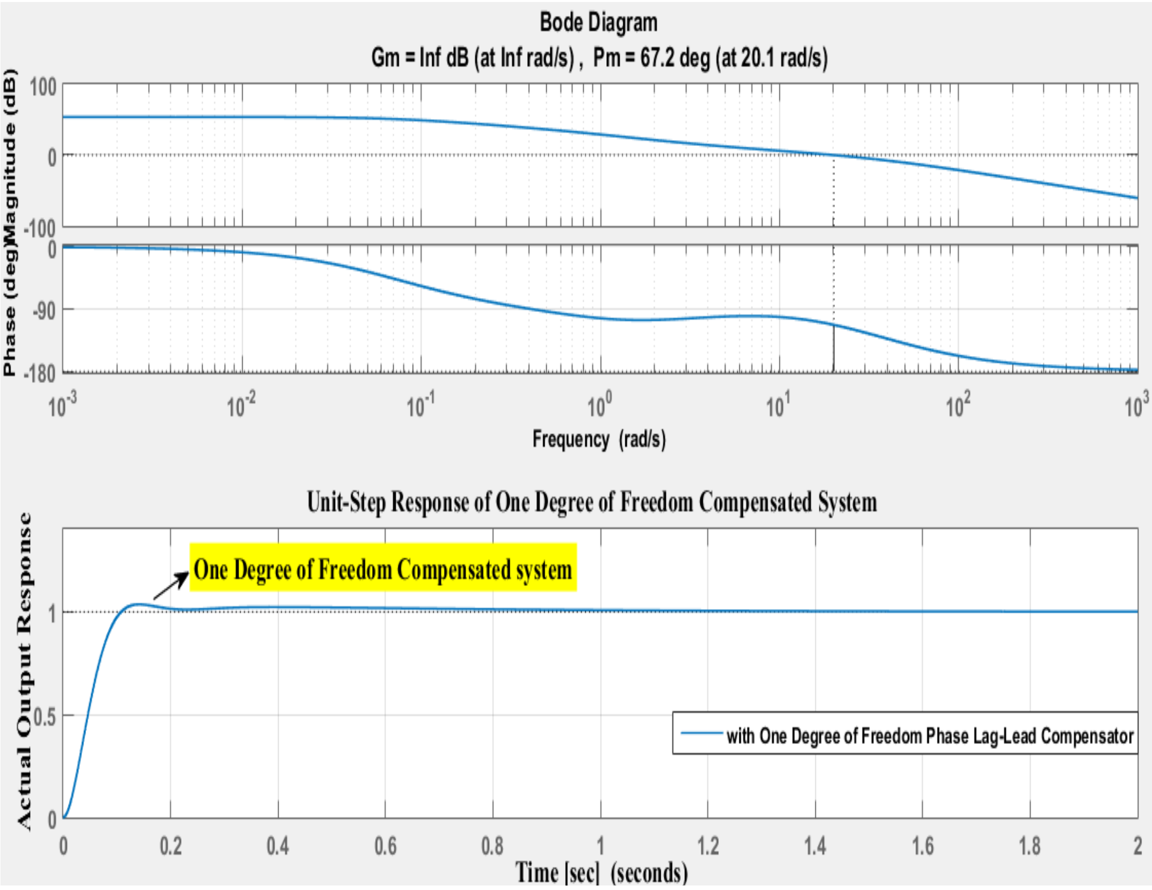

Figure 14: Simulation of the DC motor with one degree of freedom phase Lag-Lead compensator in addition to Bode plot

The block diagram of the DC motor with two degree of freedom phase Lag-Lead compensator is shown in Fig. 15.

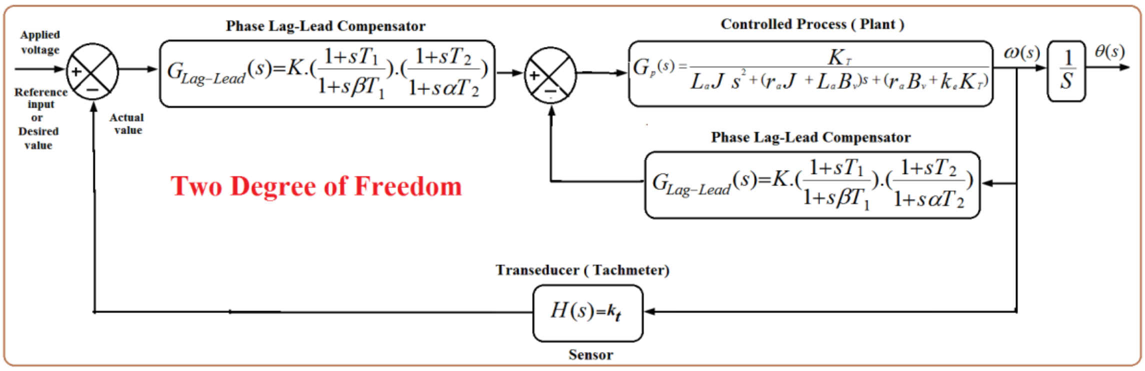

Figure 15: The block diagram of the DC motor with two degree of freedom phase Lag-Lead compensator

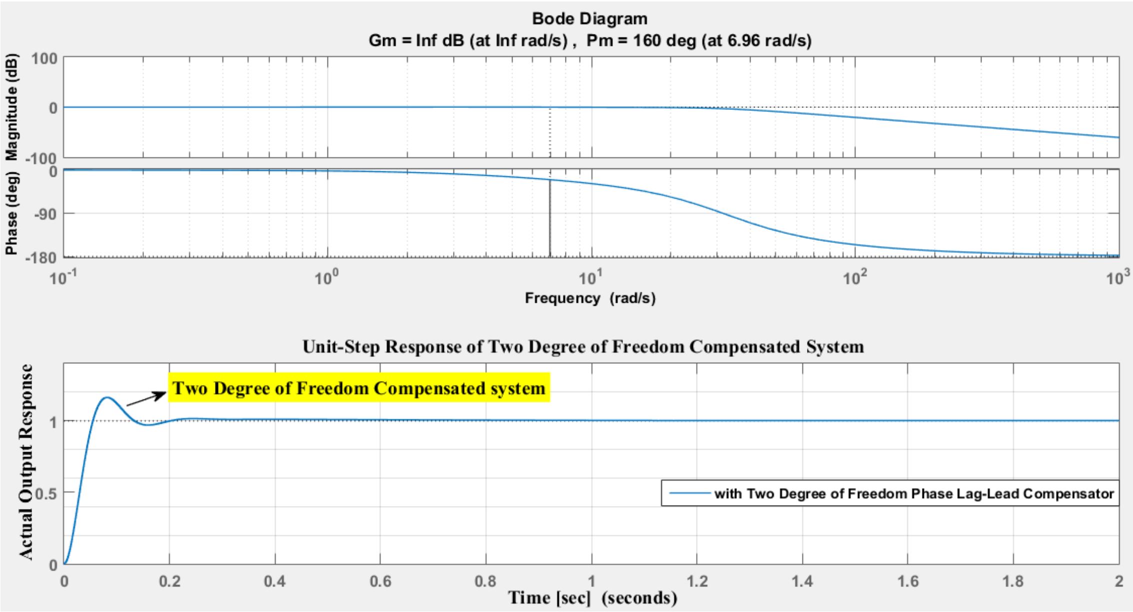

Figure 16: Simulation of the DC motor with two degree of freedom phase Lag-Lead compensator in addition to Bode plot

From the Fig. 14 and Fig. 16, it is noted that that, actual output response of the DC motor with one and two degree of freedom phase lag-lead compensator (Compensated system) is better than actual output response of the DC motor without compensator (Uncompensated system), and also all design requirements of the systems are satisfied and fulfilled) with one degree of freedom phase lag-lead compensator unlike two degree of freedom phase lag-lead compensator where some of design requirements of the systems are not satisfied and fulfilled. On the other side, two degree of freedom phase lag-lead compensator is better than one degree of freedom phase lag-lead compensator in some of design requirements of the system. but, one degree of freedom phase lag-lead compensator is chosen to drive the DC motor, because all the design requirements of the system are satisfied and fulfilled.

E. Comparison All the simulation results with design requirements of the system

In order to verify the effect of the one and two degree of freedom phase lag-lead compensator in terms of time response specifications (Damping ratio (ξ), Settling Time (ts), Maximum Overshoot (%Mp), Steady-State Error (ess), Peak Time (tp), Delay Time(td) and Rise Time (tr)), and in terms of Frequency response specifications (Phase Margin (P.M), Gain Margin (G.M), Bandwidth (ωb), Resonant Peak (Mr), Resonant Frequency (ωr), Cut-off Frequency (ωc), Gain crossover Frequency (ωgc) and Phase crossover Frequency(ωpc)), one and two degree of freedom phase lag and phase lead compensators have been compared with one and two degree of freedom phase lag-lead compensators. In addition to that, one degree of freedom phase lag-lead compensator is compared with two degree of freedom phase lag-lead compensator in terms of time response specifications and frequency response specifications.

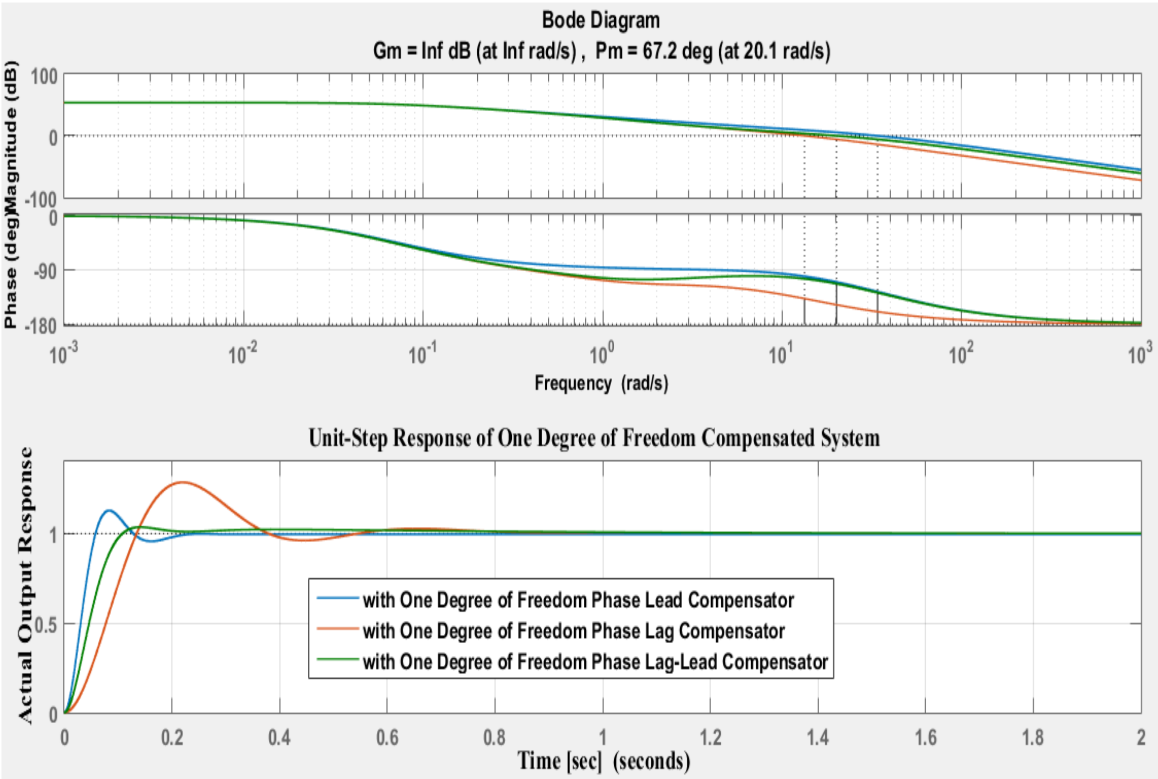

Figure 17: Simulation of the DC motor with One Degree of Freedom Phase Lead, Lag and Lag-Lead compensator in addition to Bode plot

Table 3: *A comparison of the simulation results of the DC motor with One Degree Of Freedom phase lag, lead and lag-lead compensators in terms of time response specifications*

Time Domain Specifications	Desired Results of the System	Strategy of Control		
		One Degree Of Freedom		
		Phase Lead Compensator	Phase Lag Compensator	Phase Lag-Lead Compensator
Settling Time (t_s)	< 0.5 sec	0.1564 Sec	0.5560 sec	0.545 sec
Maximum Overshoot (M_p)	< 5 %	8.4927 %	8.5210%	3.4276 %
Peak Time (t_p)	< 0.15 sec	0.097 Sec	0.3420 sec	0.141 sec
Rise Time (t_r)	< 0.1 sec	0.0456 Sec	0.1631 sec	0.0691 sec
Delay Time (t_d)	< 0.05 sec	0.0326 sec	0.118 sec	0.0462 sec
Steady state error (e_{ss})	$\leq$ 0.02	0.0020367	0.0047966	0.002
Damping ratio (ζ)	(0. 6$\leq \zeta \leq$ 0.8)	0.61745	0.61693	0.73179

Table 4: *A comparison of the simulation results of the DC motor with One Degree Of Freedom phase lag, lead and lag-lead compensators in terms of Frequency response specifications*

Frequency Domain Specifications	Desired Results of the System	Strategy of Control		
		One Degree Of Freedom		
		Phase Lead Compensator	Phase Lag Compensator	Phase Lag-Lead Compensator
Phase Margin (P.M)	Positive and , at least $\geq 60^{0}$	60.69^{0}	59.89^{0}	67.1699^{0}
Gain Margin (G.M)	Positive and large, at least $\geq$ 10 (dB)	Inf dB	Inf dB	Inf dB
Bandwidth (ω_b)	Large as can possible	46.5959 Hz	13.1257 Hz	30.9210 Hz
Resonant Peak (M_r)	Small as can possible $[1 \leq M_r \leq 1.4]$ or $[0 \text{ dB} \leq M_r \leq 3 \text{ dB}]$	1.0295	1.0298	M_r=1, if ($\zeta > 0.707$) (ζ=0.73179)
Resonant Frequency (ω_r)	Large as can possible	20.1881 Hz	5.6985 Hz	ω_r is real only if ($\zeta < 0.707$) but (ζ=0.73179)
Cut-off Frequency (ω_c)	Large as can possible	46.5959 Hz	13.1257 Hz	30.9210 Hz
Gain crossover Frequency (ω_{gc})	It must be $\omega_{gc} < \omega_{pc}$ (Hz)	29.1452 Hz	8.2090 Hz	20.1170 Hz
Phase crossover Frequency(ω_{pc})	It must be $\omega_{gc} < \omega_{pc}$ (Hz)	Inf Hz	Inf Hz	Inf Hz

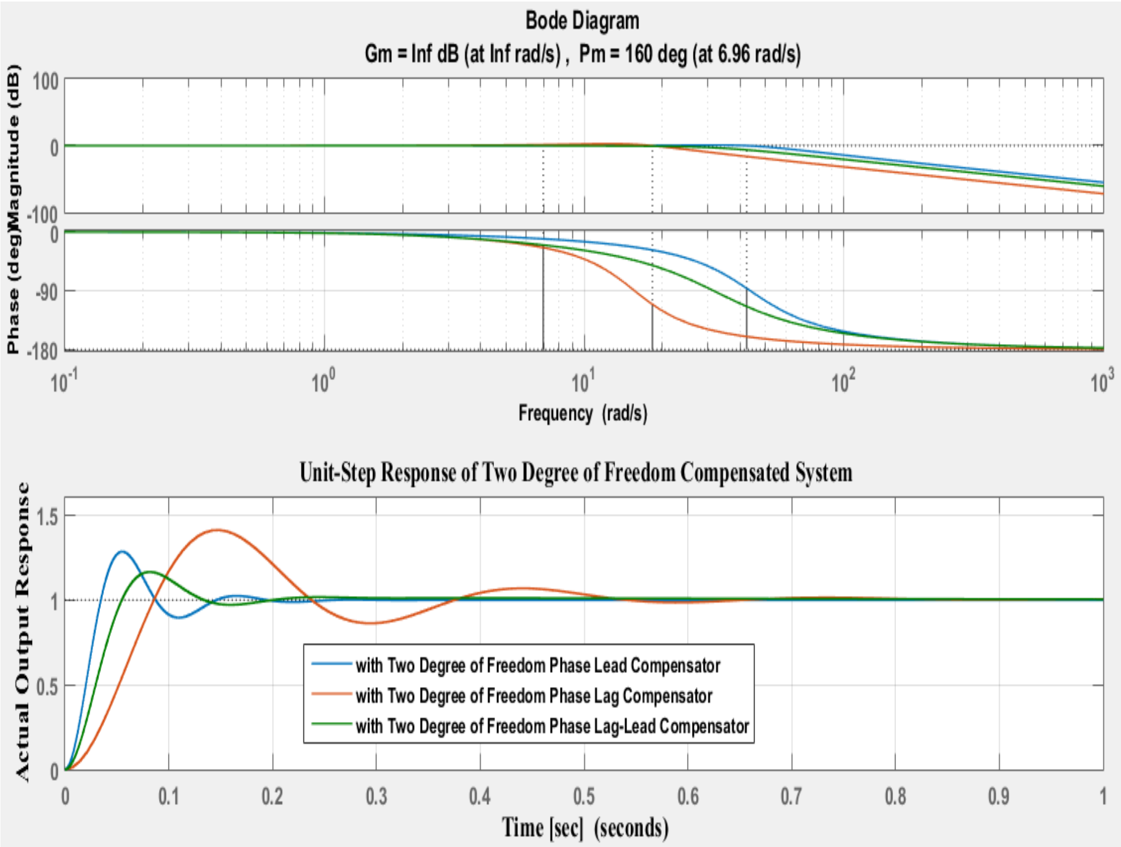

Figure 18: Simulation of the DC motor with One Degree of Freedom Phase Lead, Lag and Lag-Lead compensator in addition to Bode plot

Table 5: *A comparison of the simulation results of the DC motor with Two Degree Of Freedom phase lag, lead and lag-lead compensators in terms of time response specifications*

Time Domain Specifications	Desired Results of the System	Strategy of Control		
		Two Degree Of Freedom		
		Phase Lead Compensator	Phase Lag Compensator	Phase Lag-Lead Compensator
Settling Time (t_s)	< 0.5 sec	0.1387 Sec	0.518 sec	0.178 sec
Maximum Overshoot (M_p)	< 5 %	20.9919 %	22.1626 %	16.3384 %
Peak Time (t_p)	< 0.15 sec	0.059 Sec	0.213 sec	0.082 sec
Rise Time (t_r)	< 0.1 sec	0.0258 Sec	0.0928 sec	0.0371 sec
Delay Time (t_d)	< 0.05 sec	0.038608 Sec	0.13128 sec	0.15245 sec
Steady state error (e_{ss})	$\leq$ 0.02	0.0010194	0.0024012	0.0010194
Damping ratio (ζ)	(0. 6$\leq$ $\zeta \leq$ 0.8)	0.44499	0.43245	0.49956

Table 6: *A comparison of the simulation results of the DC motor with Two Degree Of Freedom phase lag, lead and lag-lead compensators in terms of Frequency response specifications*

Frequency Domain Specifications	**Desired Results of the System**	**Strategy of Control**		
		Two Degree Of Freedom		
		Phase Lead Compensator	**Phase Lag Compensator**	**Phase Lag-Lead Compensator**
Phase Margin (P.M)	Positive and , at least $\geq 60^0$	123.1635^0	119.4368^0	159.8746^0
Gain Margin (G.M)	Positive and large, at least $\geq$ 10 (dB)	Inf dB	Inf dB	Inf dB
Bandwidth (ω_b)	Large as can possible	45.2216 Hz	13.3348 Hz	11.2658 Hz
Resonant Peak (M_r)	Small as can possible [$1 \leq M_r \leq 1.4$] or [0 dB $\leq M_r \leq 3$ dB] $M_r=1$, if ($\zeta > 0.707$)	1.2547	1.2823	1.1554
Resonant Frequency (ω_r)	Large as can possible ω_r is real only if($\zeta < 0.707$)	26.3996 Hz	7.8512 Hz	6.2657 Hz
Cut-off Frequency (ω_c)	Large as can possible	45.2216 Hz	13.3348 Hz	11.2658 Hz
Gain crossover Frequency (ω_{gc})	It must be $\omega_{gc} < \omega_{pc}$ (Hz)	28.0024 Hz	8.2646 Hz	6.9627 Hz
Phase crossover Frequency(ω_{pc})	It must be $\omega_{pc} > \omega_{gc}$ (Hz)	Inf	Inf Hz	Inf Hz

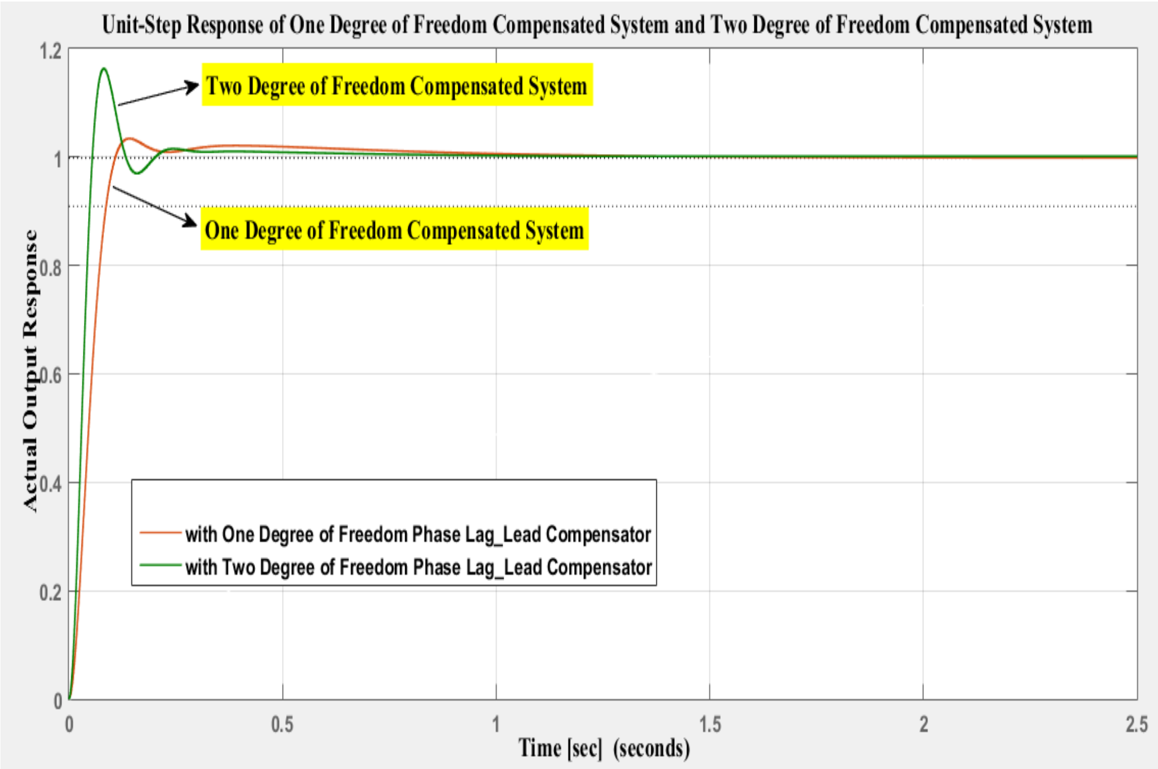

Figure 19: Simulation of the DC motor with One and Two Degree of Freedom Phase Lead, Lag and Lag-Lead compensator

Table 7: *A comparison of the simulation results of the DC motor with One and Two Degree Of Freedom phase lag, lead and lag-lead compensators in terms of time response specifications*

Time Domain Specifications	Desired Results of the System	Strategy of Control	
		One Degree Of Freedom Phase Lag-Lead Compensator	Two Degree Of Freedom Phase Lag-Lead Compensator
Settling Time (t_s)	< 0.5 sec	0.545 sec	0.178 sec
Maximum Overshoot (M_p)	< 5 %	3.4276 %	16.3384 %
Peak Time (t_p)	< 0.15 sec	0.141 sec	0.082 sec
Rise Time (t_r)	< 0.1 sec	0.0691 sec	0.0371 sec
Delay Time (t_d)	< 0.05 sec	0.0462 sec	0.15245 sec
Steady state error (e_{ss})	$\leq$ 0.02	0.002	0.0010194
Damping ratio (ζ)	($0.6 \leq \zeta \leq 0.8$)	0.73179	0.49956

The main objective of controllers is to minimize the error signal or in other words the minimization of performance criteria (Integral Absolute Error (IAE), Integral Time Absolute Error (ITAE), Integral Square Error (ISE), Integral Timeweighted Squared Error (ITSE)). A set of performance indicators may be used as a design tool aimed to evaluate tuning methods results. The simulation results are shown on Table (8).

Table 8: *Comparison for all performance indices parameters of the DC motor with One and Two Degree of Freedom Phase Lag-Lead Compensator*

Strategy of Control	Performance Criteria			
	IAE $IAE=\int_0^t \lvert e(t)\rvert\, dt$	ITAE $ITAE=\int_0^t t\lvert e(t)\rvert\, dt$	ISE $ISE=\int_0^t (e(t))^2 dt$	ITSE $ITSE=\int_0^t t(e(t))^2 dt$
One Degree of Freedom Phase Lag-Lead Compensator	**0.0239828778**	**0.0599571945**	**0.0001150357**	**0.0002875892**
Two Degree of Freedom Phase Lag-Lead Compensator	**0.00509684**	**0.0127420999**	**0.0000051956**	**0.0000129889**

V. Conclusion

The actual output response of the DC motor is controlled by means of the three different compensators: One and Two Degree of Freedom Phase lead compensator, phase lag compensator and phase lag-lead compensator for enhancement the stability and accuracy. In addition to that, the effect of adding two degree of freedom compensators on the transient and steady state response of the system is studied. In this paper, with reference to the results of the computer simulation by using (MATLAB & SIMULINK) software, the performance characteristics of One and Two Degree of Freedom Phase lead compensator, phase lag compensator and phase lag-lead compensator are compared in terms of the time response (transient and steady state response) specifications: Delay Time (td), Rise Time (tr), Peak Time (tp), Maximum Overshoot (Mp), Settling Time (ts) and Steady state error (ess), and in terms of the frequency response specifications : Phase Margin (P.M), Gain Margin (G.M), Bandwidth (ωb), Resonant Peak (Mr), Resonant Frequency (ωr), Cut-off Frequency (ωc), Gain crossover Frequency (ωgc) and Phase crossover Frequency (ωpc). The simulation results illustrate that one degree of freedom phase lag-lead compensator performs better than other one degree of freedom compensators proposed in this paper and has verified all design requirements of the system. In addition to that, one degree of freedom phase lag-lead compensator performs better than two degree of freedom phase lag-lead compensator in some design requirements of the system where two degree of freedom phase lag-lead compensator performs better than one degree of freedom phase lag-lead compensator in some design requirements of the system, but one degree of freedom phase lag-lead compensator is chosen to drive the DC motor, because all the design requirements of the system are satisfied and fulfilled unlike two degree of freedom phase lag-lead compensator where some design requirements of the system are not satisfied and fulfilled.

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