Design of PID Controller used Ziegler-Nichols technique (Robust Control)

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Abstract:

This paper studies the design of a proportional-differential-integral (PID) control for position control system using the Ziegler-Nichols (Z-N) Method.

The study begins with a complete presentation of the mathematical model of the PID control, and a presentation of the scientific steps followed in the design theory of the Ziegler-Nichols Method, then the practical application of the theory using the mathematical model of a laser manipulator that uses a DC motor Laser Manipulator Control System Where the laser can be used to drill the hip socket to insert the artificial hip joint appropriately, and the use of laser in surgery requires high accuracy in response to location and speed, and this is what the proportional-differential-integral controller works on.

The design steps using the Ziegler-Nichols Method are summarized in calculating the forward path gain Kp using the Routh method and equating the forward path gain coefficient Kp with the coefficient Kcr and substituting the coefficient Kcr in the characteristic equation to obtain Pcr. Through the table, by estimating the Ziegler-Nichols Theory, we calculate the three proportional-differential-integral controller factors Tp, Ti and Td, which determine the response of the system that was designed.

The practical part of the paper uses MATLAB to program and simulate the control system design, and to extract and analyze the results.

Study Objectives

1- Study and design of a proportional-differential-integral (PID) control to control the position control system using the Ziegler-Nichols Method.

2- Practical application of the theory on the mathematical model of a laser manipulator using a DC motor Laser Manipulator Control

3- Calculate the PID controller factors (Tp, Ti and Td), which determine the response of the designed system.

4- Use MATLAB to program and simulate the design of the control system

الملخص:

تدرس هذه الورقة تصميم متحكم تناسبي – تفاضلي – تكاملي، للتحكم بالوضع باستخدام نظرية نيكلوس –زيجلر.

تبدا الدراسة بعرض كامل للنموذج الرياضي للمتحكم تناسبي – تفاضلي – تكاملي، وعرض للخطوات العلمية المتبعة بنظرية التصميم، ثم التطبيق العملي للنظرية باستخدام النموذج الرياضي لمناور ليزر يستخدم محرك تيار مستمر، حيث يمكن استخدام الليزر لحفر مقبس الورك لإدخال مفصل الورك الاصطناعي بشكل مناسب، ويتطلب استخدام الليزر في الجراحة دقة عالية في الاستجابة للموقع والسرعة وهذا ما يعمل علية متحكم تناسبي – تفاضلي – تكاملي.

تتلخص خطوات التصميم باستخدام نظرية نيكلوس – زيجلر في حساب كسب المسار الامامي Kp باستخدام نظرية راوث ومسواة معامل كسب المسار الامامي Kp بالمعامل $\mathbf{K_{cr}}$ والتعويض بالمعامل K_{cr} بمعادلة الخواص لنتحصل على $\mathbf{P_{cr}}$ ومن خلال الجدول بتقدير نظرية نيكلوس –زيجلر، نحسب عوامل المتحكم **تناسبي** – تفاضلي – تكاملي الثلاث $\mathbf{T_d}$ and $\mathbf{T_i}$, Tp والتي تحدد استجابة النظام الذي تم تصميمة.

يستخدم بالجزء العملي للورقة برنامج ماتلاب **MATLAB** لبرمجة ومحاكاة تصميم نظام التحكم، واستخلاص النتائج وتحليلها.

أهداف الدراسة:

1– دراسة وتصميم متحكم تناسبي – تفاضلي – تكاملي للتحكم بالوضع باستخدام نظرية نيكلوس –زيجلر.

2- التطبيق العملي للنظرية على النموذج الرياضي لمناور ليزر يستخدم محرك تيار مستمر.

3- حساب عوامل المتحكم تناسبي - تفاضلي - تكاملي (T_p ,T_i and T_d)، والتي تحدد استجابة النظام الذي تم تصميمه.

4- استخدم برنامج ماتلاب **MATLAB** لبرمجة ومحاكاة تصميم نظام التحكم.

Introduction

Most industrial controllers used today operate on PID control schemes. Analog PID controllers are mostly hydraulic. Other types are pneumatic, electrical, electronic or combinations of them. Nowadays, many of them are converted to digital forms by using microprocessors. Since most PID controllers are tuned at the site of their modules, many different types of tuning rules have been designed in the literature. Using these tuning rules, precise and accurate tuning of PID controllers can be performed at the site. Also, automatic tuning methods have been developed and some PID controllers may have online automatic tuning capabilities1,2].

This paper studies the design of a proportional-differential-integral (PID) controller for position control system using the Ziegler-Nichols Method.

The study begins with a complete presentation of the mathematical model for the proportional - differential - integral controller (PID control), and a presentation of the scientific steps followed by the design theory Ziegler-Nichols Method, then the practical application of the theory using the mathematical model for a laser manipulator that uses a constant current motor Laser Manipulator Control System, where the laser can be used to drill Hip socket to insert the artificial hip joint appropriately. The use of laser in surgery requires high accuracy in responding to location and speed, and this is what a proportional-differential-integral controller works on.

The design steps using the Ziegler-Nichols Method are summarized in calculating the forward path gain Kp using Routh's theory, equating the forward path gain coefficient Kp with the factor Kcr, and replacing the factor Kcr with the characteristic equation to obtain Pcr, and through the table estimating the Nicklaus-Ziegler theory, we calculate The three control factors are proportional - differential - integral, Tp, Ti and Td, which determine the response of the system that was designed.

The practical part of the paper uses MATLAB to program and simulate the design of the control system, and to extract and analyze results.

The paper is organized as follows:control low of continues PID controller in section 1.1, in section 2.1.2 Ziegler - Nichols rules for tuning PID controllers, section 2.2.3 second method for Ziegler - Nichols rules, section 3. Simulation and Results and section 3.1 Case study of Laser Manipulator Control System

Methodology

This paper is conducted on a laser manipulator module to control a DC motor. The aim is to reduce the high overshoot time and high overshoot that the system encounters using ZN-PID, in order To improve the performance of the system outputs designed in the previous steps, the experiment and simulation of the system were conducted using Matlab programs

Design of continues PID controller

The proportional-Integral-Derivative **(PID controller**) is often referred to as a 'three-term' controller. It is currently one of the most frequently used controllers in the process industry. In a **PID controller** the control variable is generated form a term proportional to the error, a term which is the integral of the error, and a term which is the derivative of the error [1]. The effect of these parameters can be stand briefly as follows :

a. Proportional: the error is multiplied by a gain K_P. A very high gain may cause instability, and a very low gain may cause the system to drift away.

b. Integral: the integral of the error is taken and multiplied by a gain K_I . The gain can be adjusted to drive the error to zero in the required time . A too high gain may cause oscillations and a too low gain may result in a sluggish response[2,1]

c. Derivative: The derivative of the error is multiplied by a gain K_D Again, if the gain is too high the system may oscillate and if the gain is too low the response may be sluggish[2,3].

1.1 Control low of continues PID controller

Figure (1.1) show the block diagram of the classical continuous-time **PID controller.** Tuning the controller involves adjusting the parameters K_P,

K_D and K_I, in order to obtain a satisfactory response. The input-output relationship of **PID controller** can be expressed as:

$$u(t) = K_P\left[e(t) + \frac{1}{T_I}\int_0^t e(t)dt + T_D\frac{de(t)}{dt}\right] \qquad (1.1)$$

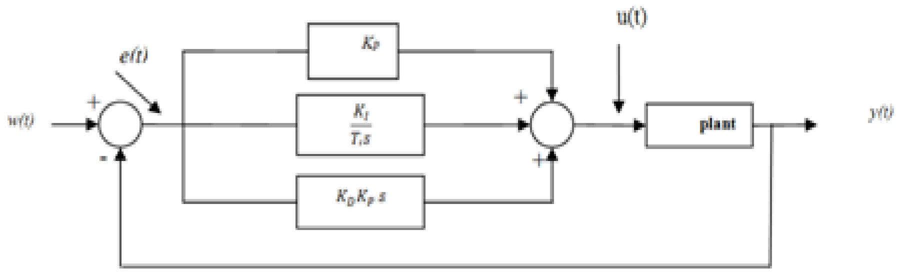
Where *u(t)* is the output from the controller and *e(t) = r(t) –y(t),* in which *r(t)* is desired set-point (reference input) and *y(t)* is the plant output.

T_I and T_D are known as the integral and derivative action time, respectively.Notice that equation (1.1) is sometimes written as:

$$u(t) = \; K_P \; e(t) + K_I \int_0^t e(t)dt + K_D\frac{de(t)}{dt} \qquad (1.2)$$

Where:

$$K_I = \frac{K_P}{T_I} \quad and \quad K_D = K_P T_D \qquad (1.3)$$



Figure(1.1) Countinuos-Time PID Control

Taking the Laplace transform of equation (1.1), we can write the transfer function of a continuous-time PID as:

$$\frac{U(s)}{E(s)} = \; K_P + \frac{K_I}{s} + K_D s \qquad (1.4)$$

To build the PID controller using a digital computer we have to convert equation (1.1) from a continuous to a discrete representation. This form of the **PID controller** is known as the velocity PID controller.

1.2 Classic PID Controllers

One form of controller widely used in industrial process control is called a three-term, or **PID controller**. This controller has a transfer function

$$G_c(s) = K_p + \frac{K_I}{s} + K_D s \qquad (1.5)$$

The controller provides a proportional term, an integration term, and a derivative term [4, 5].The equation for the output in the time domain is

$$u(t) = K_P e(t) + K_I \int e(t)\, dt + K_D \frac{de(t)}{dt} \quad (1.6)$$

The Also called a tri-mode controller a **PID** controller because it contains a proportional, an integral, and a derivative term, Figure (1.1) show the block diagram of the Three-Term **PID controller**.

Consider the PID controller the closed-loop trans- fer function is[5,6] :

$$\mathrm{T(s)} = \frac{G(S)\, G_c(S)}{1+G(S)G_c(S)} \quad (1.7)$$

2. Tuning method of Continues PID controller

PID Control in Processes. Figure (2.1) shows a PID controller in an industrial process. When a mathematical model of the industrial process can be derived, it becomes easy to apply various theories to design and specify the controller parameters that will adjust the transient and steady state specifications of the closed loop system. However, if the system is so complex that its mathematical model cannot be easily obtained, an analytical approach to designing a PID controller will not be possible

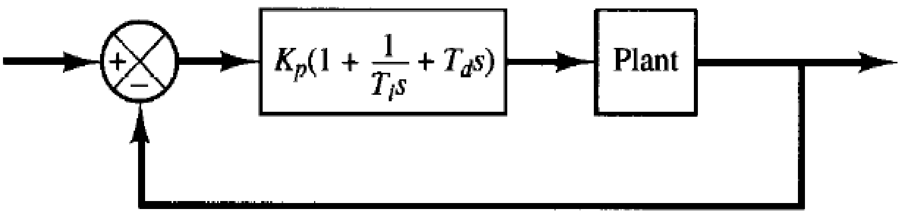


Figure (2.1) PID control of a plant

. Then we must resort to experimental approaches to the tuning of PID controllers[4].

2.1 PID Control Design used Z-N Method

The process of selecting controller parameters to optimize a given performance specification is known as controller tuning. Z-N has proposed rules for tuning PID controllers (i.e., selecting values of Kp, Ti, and Td) based on the experimental step response or on the value of Kp that results in marginal stability when using only proportional control procedure. Z-N rules, which are presented in the following, are very convenient when mathematical models of plants are not known. (These rules can, of course, be applied to the design of systems with known mathematical models)

2.1.1 Z-N rules for PID parameters tuning

Ziegler and Nichols proposed method for determining values of the proportional gain $\mathbf{K_p}$, integral time $\mathbf{T_i}$ and derivative time $\mathbf{T_d}$ based on the transient response characteristics of a given plant. Such determination of the parameters of PID controllers or tuning of PID controllers can be made by engineers on site by experiments on the plant. (Numerous tuning rules for PID controllers have been proposed since the Z-N proposal. They are available in the literature. However, here we consider only the Z-N tuning laws.) There are two methods called Z-N tuning method. In both methods, they aimed at obtaining 25% maximum overshoot in step response (see Figure 2.2). this paper will examine the second theory only.

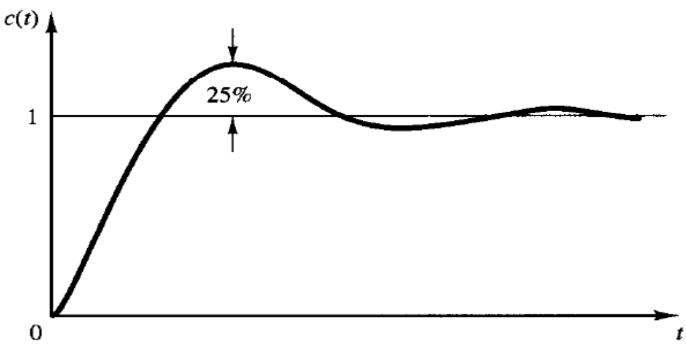


Figure (2.2) Unit-Step Response Curve

2.2 Second Method for Z-N rules

In the second theorem, first Ti=∞ and Td=0 are set by the proportional control procedure only (see Fig. 2.3), we increase Kp, from 0 to the critical value Kcr, where the output first produces continuous oscillations. (If the output does not produce continuous oscillations for any value that Kp may take, this method is not applicable.) Here, the critical gain Kcr and the corresponding period Pcr are determined experimentally, see Fig. (2.4). Z-N suggested that we adjust the values of the parameters K, T, and 7 according to the value shown in Table (1.1), Theorem 2 [4].

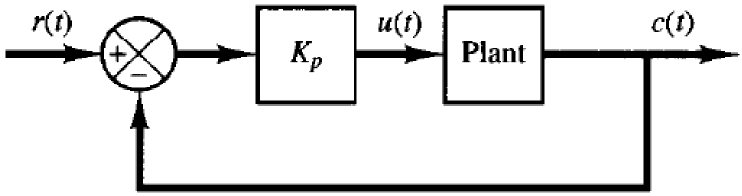
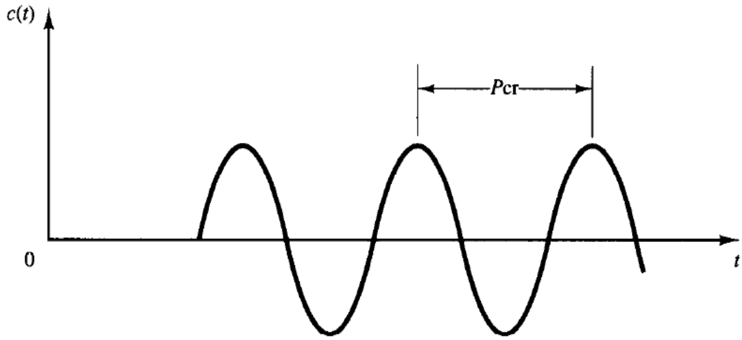


Figure (2.3) Closed Loop System with Proposnal Controller



Figure(2.4)Sustaned Oscillation witn Period P_{cr}

Table 1.1 Ziegler Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period $\mathbf{P_{cr}}$

Type of controller	K_p	T_i	T_d
P	**$0.5K_{cr}$**	∞	**0**
PI	**0.45Kcr**	$\frac{1}{1.2} P_{cr}$	**0**
PID	**$0.6K_{cr}$**	**$0.5P_{cr}$**	**$0.125P_{cr}$**

That the PID controller tuned by the second method of Ziegler-Nichols rules gives

$$G_C(s) = K_p\left(1+\frac{1}{T_i s}+T_d s\right) \quad (2.1)$$

$$= 0.6\text{Kcr}\left(1+\frac{1}{0.5\text{Pcr}s}+0.125\text{Pcr}\right) \quad (2.2)$$

$$= 0.075\text{KcrPcr}\,\frac{(s+\frac{4}{\text{Pcr}})^2}{s} \qquad (2.3)$$

Thus, the PID controller has a pole at the origin and double zeros at s= -4/P.

Z-N tuning rules are widely used to tune PID controllers in unit control systems where unit dynamics are not precisely known. Z-N tuning rules can of course be applied to industrial units whose dynamics are known. (Once unit dynamics are known, there are many analytical and graphical methods for designing PID controllers, in addition to Z-N tuning rules. If the transfer function of the industrial unit is known, the unit step response can be calculated or the critical gain Kcr and critical period Pcr can be calculated. Using these calculated values, it is easy to determine the parameters Kp, Ti, and Td from Table (1.1). However, the real benefit of the Ziegler-Nichols (and other) tuning rules becomes apparent when the dynamics of the industrial unit are not known so that no analytical or graphical methods are available for designing the controllers.

Simulation and Results

Case study : Laser Manipulator Control System

Laser is used to drill the hip cavity for precise and appropriate insertion of the artificial hip joint. It is important for the use of laser in surgery to have high accuracy in response to position and speed. See the system shown in Figure (3.1). It uses a DC motor for the laser manipulator [5,6].

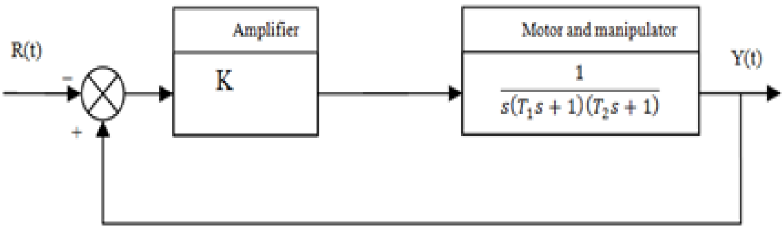


Figure (3.1)Laser Manipulator Control System

We will use the PD controller later in this paper to control the design of the laser manipulator control system

The amplifier gain K must be adjusted so that the steady-state error for a ramp input $r(t)=At$ (where $A=1$ mm/s), is less than or equal to 0.1mm, while a stable response is maintained.

To obtain the steady-state error required and a good response. We select a motor with a field time constant T_1=0.1s and a motor-plus-load time T_2=0.2s. we then have:

$$\frac{KG(s)}{1+KG(s)} = \frac{K}{s(T_1 s+1)(T_2 s+1)+K} \quad (3.1)$$

$$\frac{K}{0.02s^3+0.3s^2+s+K} = \frac{50K}{s^3+15s^2+50s+50K}$$

The steady-state error for a ramp. R(s)=A/s^2, is :

$$e_{ss} = \frac{A}{K_v} = \frac{A}{K} \quad (3.3)$$

Since we desire e_{ss}=0.1 mm (or less) and A=1 mm. we require K =10 (or greater).

To ensure a stable system, we obtain the characteristic equation from Equation (3.2) as :

$$s^3+15s^2+50s+50K \quad (3.4)$$

Establishing the Routh array, we have

$$\begin{array}{c|cc} s^3 & 1 & 50 \\ s^2 & 15 & 50K \\ s^1 & b_1 & 0 \\ s^0 & 50K & - \end{array}$$

Where:

$$b = \frac{750-50K}{15}$$

therefore, the system is stable for $0 \le K \ge 15$

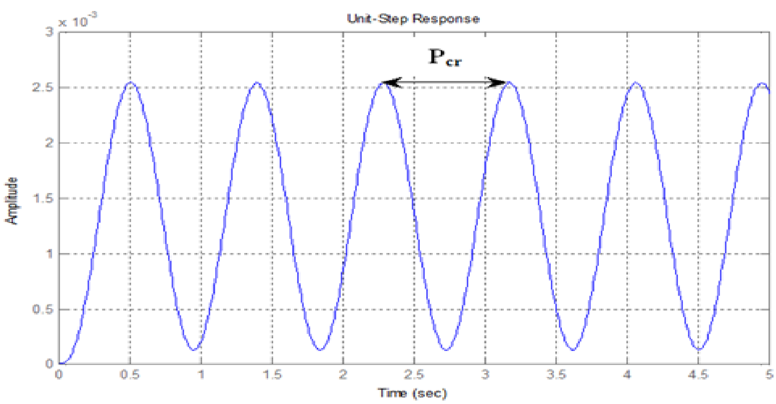


Figure (3.2) Sustained Osscillation with Period P_{cr}

$k = K_{cr} = 15$. See Figure (3.2), the critical gain K_{cr} and the corresponding period $\mathbf{P_{cr}}$ are experimentally determined .
With gain K set equal to K_{cr} =15, the characteristic equation becomes

$$15s^3 + s^2 + 50s + 750 = 0$$

To find the frequency of the sustained oscillation, we substitute s =jω into this characteristic equation as follows: $(j\omega)^3 + 15\,(j\omega)^2 + 50(j\omega) + 750 \; = 0$

$$15(50 - \omega^2) + j\omega(50 - \omega^2) = 0$$

$$\omega^2 = \;\; 50$$

or $\omega = \sqrt{50}$

$$P_{cr} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{50}} = 0.8881$$

Referring to table (1.1) determine K_p, $\mathbf{T_i}$ and $\mathbf{T_d}$ as follows:

$$\boldsymbol{KP} \; = 0.6K_{cr} \; = 9 \qquad (3.5)$$

$$\boldsymbol{T_i} \; = \; 0.5P_{cr} \; = \; 0.444 \qquad (3.6)$$

$$\boldsymbol{T_d} \; = 0.125P_{cr} \; = \; 0.111 \qquad (3.7)$$

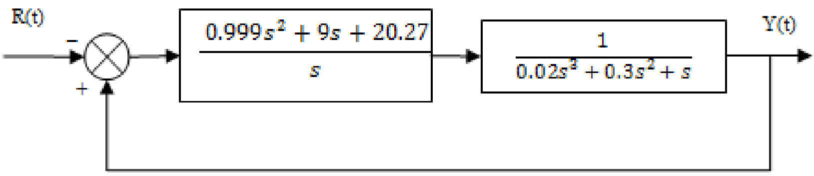
$$G_C(s) = \; K_p(1 + \frac{1}{T_i s} + T_d s)$$

$$= 9\left(1 + \frac{1}{0.444\text{s}} + 0.111\text{s}\right) = 9(\frac{0.444\text{s} + 1 + 0.049284\text{s}^2}{0.444\text{s}})$$

$$= \frac{0.999s^2 + 9s + 20.27}{s} = 0.999\left(\frac{s^2 + 9s + \; 20.29}{s}\right)$$

The PID controller has a pole at the orgigin at S= -9+j0.2 , S= -9-j0.2 . Ablock diagram of the control system with the designed PID controller is shown in Figure (3.3)The closed-loop transfer function $\frac{C(s)}{R(s)}$ is given by:

$$\frac{C(s)}{R(s)} = \frac{49.98s^2 + 450s + 1012.5}{s^4 + 15s^3 + 99.98s^2 + 450s + 1012.5} \quad (3.8)$$



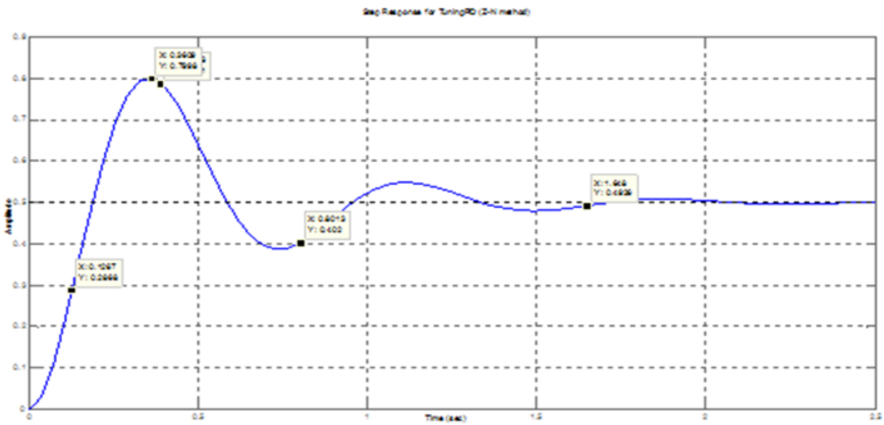
Figure(3.3)Block diagram of the system with PID controller designed by use of Ziegler-Nichols tuning rule

Resultes

The resulting unit-step response curve is shown in Figure(3.4).The unit-step response of this system can obtained easily with MATLAB. The maximum overshoot in the unit step response is approximately 60%. The amount of maximum oversoot is excessive. It can be reduced by fine tuning the controller parameters.The Time Domain is shown in Table(3.1)[7].

Table (3.1) Time Domain Spcification

Controller	**Time Domain**					
PID	**RiseTime**	**SettlingTime**	**Settling Max**	**OverShoot**	**Peak**	**PeakTime**
	0.1274	**1.6265**	**0.8003**	**60%**	**0.8**	**0.3635**



Figure(3.4):Unit-step Response Curve with PID Controller

Conclusion

The main problem in position control systems is achieving a precisely defined position. In this paper, we use a controller PID (Proportional-Integral-Derivative), which determines the difference between the desired value and the actual value of the position, and then adjusts the control signal based on this difference. The parameters of the PID controller are adjusted in a way that allows achieving stability and accuracy in the system. The paper studies the design of the controller PID using a mathematical model. The control parameters of the PID controller are adjusted using the theory of the Ziegler-Nichols Method . we calculate the fornt path gain, K using Routh method. Replace the factor K_{cr} with the characteristic equation to obtain P_{cr} and through the table by estimating Ziegler-Nichols, we calculate the PID parameters Ti, T_d, T_p with determines the response of the system that was designed and by applying of the step response using the laser Manipulaor model and by applying the step function to the system, we obtain the required time response. To find the Simulation of the system design with the time response, we use a MATLAB programming.

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